



Date :- 01/08/2024

NOTIFICATION

Sub :- CBCS Syllabi of B. Sc. in Mathematics (Sem. I & II)

Ref. :- Decision of the Academic Council at its meeting held on 27/07/2024.

The Syllabi of B. Sc. in Mathematics (First and Second Semesters) as per **NATIONAL EDUCATION POLICY – 2020 (2024 Pattern)** and approved by the Academic Council as referred above are hereby notified for implementation with effect from the academic year 2024-25.

Copy of the Syllabi Shall be downloaded from the College Website
(www.kcesmjcollege.in)

Sd/-
Chairman,
Board of Studies

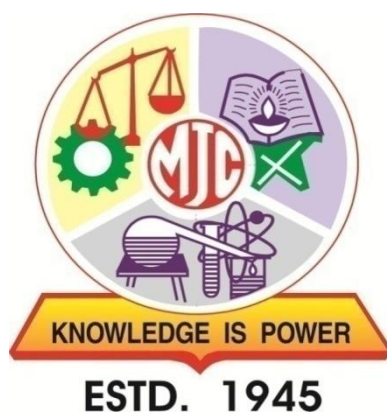
To :

- 1) The Head of the Dept., M. J. College, Jalgaon.
- 2) The office of the COE, M. J. College, Jalgaon.
- 3) The office of the Registrar, M. J. College, Jalgaon.

Knowledge is Power

Khandesh College Education Society's
Moolji Jaitha College, Jalgaon
An "Autonomous College"

Affiliated to
Kavayitri Bahinabai Chaudhari
North Maharashtra University, Jalgaon-425001



STRUCTURE AND SYLLABUS

B.Sc. Honours/Honours with Research (F.Y.B.Sc. Mathematics)

**Under Choice Based Credit System (CBCS)
and
as per NEP-2020 Guidelines**

[w.e.f.Academic Year:2024-25]

Preface

The Moolji Jaitha College (Autonomous) has adopted a department-specific model as per the guidelines of UGC, NEP-2020 and as per notification of Government of Maharashtra dt. 13th March 2024. The Board of Studies in Mathematics of the college has prepared the syllabus for the first-year graduate of Mathematics. The syllabus cultivates theoretical knowledge and applications of different fields of Mathematics. The contents of the syllabus have been prepared to accommodate the fundamental aspects of various disciplines of Mathematics and to build the foundation for various applied sectors of Mathematics. The program will be enlightened the students with the advanced knowledge of Mathematics, which will help to enhance student's employability.

The overall curriculum of three/four year covers pure mathematics, applied mathematics and computational mathematics with programming. The syllabus is structured to cater the knowledge and skills required in the research field, Industrial Sector and Entrepreneurship etc.. The detailed syllabus of each paper is appended with a list of suggested readings.

Programme Outcomes (PO) for B.Sc. honours/ Honours with Research

Upon successful completion of the B.Sc. program, student will be able to:

PO No.	PO
1	Understand the basic concepts and fundamental principles related to various science branches
2	Aquaintthe skills in handling scientific instruments and performing in laboratory experiments
3	Relate various scientific phenomena and their relevancies in the day-to-day life.
4	Analyse experimental data critically and systematically to draw the objective conclusions.
5	Develop various skills such as communication, leadership, teamwork, social, research etc., which will help in expressing ideas and views clearly
6	Develop interdisciplinary approach for providing better solutions and sustainable developments.

Programme Specific Outcome (PSO)for B.Sc. Mathematics Honours/Honours with Research:

After completion of this course, students are expected to:

PSO No.	PSO
1	Demonstrate the concepts involved in Real analysis, Matrix Theory, Differential equations, Algebra, Number Theory and Applied Mathematics.
2	Gain proficiency in mathematical techniques of both pure and applied mathematics and will be able to apply necessary mathematical methods to a scientific problem.
3	Acquire significant knowledge on various aspects related to Linear algebra, Metric spaces, Lattice theory, Integral transforms, Optimization techniques and Partial Differential equations.
4	Learn to work independently as well as a team to formulate appropriate mathematical methods.
5	Develop the ability to understand and practice the morality and ethics related to scientific research.
6	Realize the scope of Mathematics and plan continue their education as a Post-Graduate student of Mathematics and contribute to Mathematics through their research as a doctoral student.

Credit distribution structure for Three/ Four year Honors/ Honors with Research Degree Programme with Multiple Entry and Exit

F.Y. B.Sc.

Year (Level)	Sem	Subject-I (M-1)	Subject-II (M-2)	Subject-III (M-3)	Open Elective (OE)	VSC, SEC (VSEC)	AEC, VEC, IKS	CC, FP, CEP, OJT, RP	Cumulative Credits/Sem	Degree/ Cumulative Credit
I (4.5)	I	DSC-1(2T) DSC-2(2P)	DSC-1(2T) DSC-2(2P)	DSC-1(2T) DSC-2(2P)	OE-1(2T)	---	AEC-1(2T) (Eng) VEC-1(2T) (ES) IKS(2T)	CC-1(2T)	22	UG Certificate
	II	DSC-3(2T) DSC-4(2P)	DSC-3(2T) DSC-4(2P)	DSC-3(2T) DSC-4(2P)	OE-2(2T) OE-3(2P)	---	AEC-2(2T) (Eng) VEC-2(2T) (CI)	CC-2(2T)	22	
	Cum. Cr.	8	8	8	6	---	10	4	44	
Exit option: Award of UG Certificate with 44 credits and an additional 4 credits core NSOF course/ Internship OR Continue with Major and Minor.										

S.Y. B.Sc.

Year (Level)	Sem	Subject-I (M-1) Major*		Subject-II (M-2) Minor #	Subject-III (M-3)	Open Elective (OE)	VSC, SEC (VSEC)	AEC, VEC, IKS	CC, FP, CEP, OJT/Int/RP	Cumulative Credits/Sem	Degree/ Cumulative Credit
		Mandatory (DSC)	Elective (DSE)	(MIN)							
2 (5.0)	III	DSC-5(2T) DSC-6(2T) DSC-7(2P)	---	MIN-1(2T) MIN-2(2T) MIN-3(2P)	---	OE-4(2T)	SEC-1(2T)	AEC-3(2T) (MIL)	CC-3(2T) CEP(2)	22	UG Diploma
	IV	DSC-8(2T) DSC-9(2T) DSC-10(2P)	---	MIN-4(2T) MIN-5(2P)	---	OE-5(2T)	SEC-2(2T) SEC-2(2P)	AEC-4(2T) (MIL)	CC-4(2T) ⊗ FP(2)	22	
	Cum . Cr.	12	---	10	---	4	6	4	8	44	
Exit option: Award of UG Diploma in Major and Minor with 88 credits and an additional 4 credits core NSQF course/ Internship OR Continue with Major & Minor.											

* Student must choose one subject as a Major subject out of M-1, M-2 and M-3 that he/she has chosen at First year

Student must choose one subject as a Minor subject out of M-1, M-2 and M-3 that he/she has chosen at First year (Minor must be other than Major)

⊗ OJT/Internship/CEP should be completed in the summer vacation after 4th semester

T.Y. B.Sc.

Year (Level)	Sem	Subject-I (M-1) Major		Subject-II (M-2) Minor	Subject-III (M-3)	Open Elective (OE)	VSC, SEC (VSEC)	AEC, VEC, IKS	CC, FP, CEP, OJT/Int/RP	Cumulative Credits/Sem	Degree/ Cumulative Credit
		Mandatory (DSC)	Elective (DSE)	(MIN)							
3 (5.5)	V	DSC-11(2T) DSC-12(2T) DSC-13(2T) DSC-14(2P) DSC-15(2P)	DSE-1A/B (2T) DSE-2A/B (2P)	---	---	---	VSC-1(2T) VSC-2(2P)	---	OJT/Int (4)	22	UG Degree
	VI	DSC-16(2T) DSC-17(2T) DSC-18(2T) DSC-19(2T) DSC-20(2T) IKS DSC-21(2P) DSC-22(2P)	DSE-3A/B (2T) DSE-4A/B (2P)	---	---	---	VSC-3(2T) VSC-4(2P)	---	---	22	
	Cum . Cr.	24	8	---	---	---	8	---	4	44	
		Exit option: Award of UG Degree in Major with 132 credits OR Continue with Major and Minor									

Fourth Year B.Sc. (Honours)

Year (Level)	Sem	Major Core Subjects		Research Methodology (RM)	VSC, SEC (VSEC)	OE	AEC, VEC, IKS	CC, FP, CEP, OJT/Int/RP	Cumulative Credits/Sem	Degree/ Cumulative Credit
IV (6.0)	VII	DSC-23(4T) DSC-24(4T) DSC-25(4T) DSC-26(2P)	DSE-5A/B (2T) DSE-6A/B (2P)	RM(4T)	---	---	---	---	22	UG Honours Degree
	VIII	DSC-27(4T) DSC-28(4T) DSC-29(4T) DSC-30(2P)	DSE-7A/B (2T) DSE-8A/B (2P)	---	---	---	---	OJT/Int (4)	22	
	Cum. Cr.	28	8	4	---	---	---	4	44	
Four Year UG Honors Degree in Major and Minor with 176 credits										

Fourth Year B.Sc. (Honours with Research)

Year (Level)	Sem	Major Core Subjects		Research Methodology (RM)	VSC, SEC (VSEC)	OE	AEC, VEC, IKS	CC, FP, CEP, OJT/Int/RP	Cumulative Credits/Sem	Degree/ Cumulative Credit
IV (6.0)	VII	DSC-23(4T) DSC-24(4T) DSC-26(2P)	DSE-5A/B (2T) DSE-6A/B (2P)	RM(4T)	---	---	---	RP(4)	22	UG Honours with Research Degree
	VIII	DSC-27(4T) DSC-28(4T) DSC-30(2P)	DSE-7A/B (2T) DSE-8A/B (2P)	---	---	---	---	RP(8)	22	
	Cum. Cr.	20	8	4	---	---	---	12	44	
Four Year UG Honours with Research Degree in Major and Minor with 176 credits										

Sem- Semester, **DSC-** Department Specific Course, **DSE-** Department Specific Elective, **OE/GE-** Open/Generic elective, **VSC-** Vocational Skill Course, **SEC-** Skill Enhancement Course, **VSEC-** Vocation and Skill Enhancement Course, **AEC-** Ability Enhancement Course, **IKS-** Indian Knowledge System, **VEC-** Value Education Course, **T-** Theory, **P-** Practical, **CC-**Co-curricular **RM-** Research Methodology, **OJT-** On Job Training, **FP-** Field Project, **Int-** Internship, **RP-** Research Project, **CEP-** Community Extension Programme, **ENG-** English, **CI-** Constitution of India, **MIL-** Modern Indian Language

- Number in bracket indicate credit
- The courses which do not have practical 'P' will be treated as theory 'T'
- If student select subject other than faculty in the subjects M-1, M-2 and M-3, then that subject will be treated as Minor subject, and cannot be selected as Major at second year.

Details of F. Y. B. Sc. (Mathematics)

Course	Course Type	Course Code	Course Title	Credits	Teaching Hours/ Week			Marks			
					T	P	Total	Internal		External	
								T	P	T	P
Semester I, Level – 4.5											
DSC-1	DSC	MTH-DSC-111	Calculus-I	2	2	---	2	20	---	30	---
DSC-2	DSC	MTH -DSC-112	Practical Course on Matrix Algebra	2	---	4	4	---	20	---	30
OE-1	OE	MTH -OE-111	Mathematics for Competitive Exams-I	2	2	---	2	20	---	30	---
Semester II, Level – 4.5											
DSC-3	DSC	MTH -DSC-121	Theory of Equations	2	2	---	2	20	---	30	---
DSC-4	DSC	MTH -DSC-122	Practical Course on Graph Theory	2	---	4	4	---	20	---	30
OE-2	OE	MTH -OE-121	Mathematics for Competitive Exams-II	2	2	---	2	20	---	30	---
OE-3	OE	MTH -OE-122	Practical Course on Mathematics for Competitive Exams	2	---	4	4	---	20	---	30

Examination Pattern

Theory Question Paper Pattern:

- 30 (External) +20 (Internal) for 2 credits
 - External examination will be of 1½ hours duration
 - There shall be 3 questions: Q1 carrying 6 marks and Q2, Q3 carrying 12 marks each. The tentative pattern of question papers shall be as follows;
 - Q1 Attempt any 2 out of 3 sub-questions; each 3 marks
 - Q 2 and Q3 Attempt any 3 out of 4 sub-question; each 4 marks.

Rules of Continuous Internal Evaluation:

The Continuous Internal Evaluation for theory papers shall consist of two methods:

1. Continuous & Comprehensive Evaluation (CCE): CCE will carry a maximum of 30% weightage (30/15 marks) of the total marks for a course. Before the start of the academic session in each semester, the subject teacher should choose any three assessment methods from the following list, with each method carrying 10/5 marks:

- i. Individual Assignments
- ii. Seminars/Classroom Presentations/Quizzes
- iii. Group Discussions/Class Discussion/Group Assignments
- iv. Case studies/Case lets
- v. Participatory & Industry-Integrated Learning/Field visits
- vi. Practical activities/Problem Solving Exercises
- vii. Participation in Seminars/Academic Events/Symposia, etc.
- viii. Mini Projects/Capstone Projects
- ix. Book review/Article review/Article preparation
- x. Any other academic activity
- xi. Each chosen CCE method shall be based on a particular unit of the syllabus, ensuring that three units of the syllabus are mapped to the CCEs.

2. Internal Assessment Tests (IAT): IAT will carry a maximum of 10% weightage (10/5 marks) of the total marks for a course. IAT shall be conducted at the end of the semester and will assess the remaining unit of the syllabus that was not covered by the CCEs. The subject teacher is at liberty to decide which units are to be assessed using CCEs and which unit is to be assessed on the basis of IAT. The overall weightage of Continuous Internal Evaluation (CCE + IAT) shall be 40% of the total marks for the course. The remaining 60% of the marks shall be allocated to the semester-end examinations. The subject teachers are required to communicate the chosen CCE methods and the corresponding syllabus units to the students at the beginning of the semester to ensure clarity and proper preparation.

Practical Examination Credit 2: Pattern (30+20)

External Practical Examination (30 marks):

- Practical examination shall be conducted by the respective department at the end of the semester.
- Practical examination will be of 3 hours duration and shall be conducted as per schedule.
- Practical examination shall be conducted for 2 consecutive days for 2 hr/ day where incubation conditionis required.
- There shall be 05 marks for journal and viva-voce. Certified journal is compulsory to appear for practical examination.
- External practical examination of SEC will be of 25 marks and there will be no internal exam for SEC practical.

Internal Practical Examination (20 marks):

- Internal practical examination of 10 marks will be conducted by department as per schedule given.
- For internal practical examination student must produce the laboratory journal of practicals completed along with the completion certificate signed by the concerned teacher and the Head of the department.
- There shall be continuous assessment of 30 marks based on student performance throughout the semester. This assessment can include quizzes, group discussions, presentations and other activities assigned by the faculty during regular practicals. For details refer internal theory examination guidelines.
- Finally 40 (10+30) marks performance of student will be converted into 20 marks.

SEMESTER-I

F.Y. B.Sc. Mathematics (Major)
Semester-I
MTH-DSC-111: Calculus-I

Total Hours: 30

Credits: 2

Course Objectives	- To know the concept of real numbers, supremum, infimum and completeness of real numbers. - Assimilate the notion of limit of a sequence. - Assimilate the notion of convergence of a series of real numbers. - To know the limit of a function at a point.	
Course Outcomes	After successful completion of this course, students are expected to: - Understand the notions of real numbers, supremum, infimum and completeness of real numbers. - Understand the notion of limit of a sequence. - Understand the notion of convergence of a series of real numbers. - Calculate the limit of a function at a point.	
Unit	Content	Hours
Unit I	Real Numbers: Algebraic properties of $\mathbb{R}$, Order properties of $\mathbb{R}$, Well-Ordering Property of $\mathbb{N}$, Arithmetic mean-Geometric mean inequality, Bernoulli's inequality. (Revision: essential properties should be revised with illustrative examples), Absolute value function and its properties, Triangle inequality and its consequences, neighbourhood of a point on a real line, Definitions of Upper bound, Lower bound, bounded sets, supremum, infimum of subsets of $\mathbb{R}$, completeness property of $\mathbb{R}$, Archimedean property and its consequences, the density theorem (without proof), Intervals of real line, nested interval property (statement only).	7
Unit II	Sequences: Definition and examples of sequences of real numbers, Definition of limit of sequence and uniqueness of limit, Examples of limit of sequence, Definition of bounded sequence, every convergent sequence is bounded, Algebra of limits, Monotone sequences, Monotone convergence theorem, Definition of subsequence and examples, Divergence criteria, Monotone Subsequence theorem (without proof), Bolzano-Weierstrass theorem (first proof), Cauchy sequence (definition and examples only).	8
Unit III	Series: Definition, Sequence of partial sums, Convergent series and Divergent series, Some tests for convergence of series (statements and examples only).	7
Unit IV	Limits: Functions and their Graphs, Definition of cluster point and examples, Definition of limit of a function, Sequential criterion for limits, Divergence criteria, Algebra of limits (proofs using sequential criterion), Squeeze theorem for limit, one sided limits, infinite limits (without proof).	8

Study Resources	• Bartle,R.G., and Sherbert,D.R., <i>Introduction to Real Analysis</i>(4thed.). John Wiley and SonsInc. (Ch.2: Art.2.1-2.5, Ch.3: Art.3.1-3.7, Ch.4: Art.4.1-4.3) • Malik, S. C. (2011).<i>Principles of Real Analysis</i>(2nded.).New Academic Science. • Howard A., Bivens,I., and Stephan D. (2016). <i>Calculus</i> (10thed.). WileyIndia. • Gabriel, K. (1986). <i>Aspects of Calculus</i>. Springer-Verlag. • Wieslaw, K.,andRai B. (2003). <i>Calculus with Maple Labs</i>. Narosa. • Prasad,G.(2016). <i>Differential Calculus</i> (19th ed.). Pothishala Pvt. Ltd. • Thomas, G. B., Hass, J., Heil,C., andWeir M. D. (2018). <i>Thomas' Calculus</i> (14th ed.). Pearson Education.	
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F.Y. B.Sc. Mathematics (Major)
Semester-I
MTH-DSC-112: Practical Course on Matrix Algebra

Total Hours: 60

Credits: 2

Course objectives	- To know the basic need of this course is to understand the concepts and applications of matrices. - It will improve problem solving and logical thinking abilities of the students. - To study the concepts of theory of matrices in linear algebra. - To use theory of matrices in solving linear equations.
Course outcomes	Upon successful completion of this course the student will be able to: - Understand operations on matrices. - Understand the concept of rank of a matrix and inverse of a matrix. - Understand the concept of eigenvalues and eigenvectors. - To use theory of matrices to scaling & shearing, reflection, rotation & translation.
Practical No.	Content
1	Adjoint of a Matrix - Elementary operations on matrices - Adjoint of a matrix
2	Inverse of a Matrix - Inverse of a matrix - Existence & uniqueness theorem of inverse of a matrix - Properties of inverse of a matrix
3	Elementary Matrices and Rank of Matrix - Elementary matrices - Rank of a matrix
4	Normal form and Rank of Matrix - Normal form of a matrix - Reduction of a matrix to its normal form - Rank of product of two matrices
5	System of Linear Equations-I - A homogeneous and non-homogeneous system of linear equations - Consistency of system of linear equations
6	System of Linear Equations-II - A homogeneous and non-homogeneous system of linear equations - Consistency of system of linear equations
7	Eigen Values - Characteristic equation - Eigen values of matrices
8	Cayley-Hamilton Theorem and Eigen Vectors - Eigen vectors of matrices - Cayley Hamilton theorem (statement only) - Inverse of a Matrix by using Cayley Hamilton theorem

Study Resources	- Iyengar, N. Ch. S. N. (1998). <i>Matrices</i>. Anmol Publications Pvt. Ltd. (Unit:IV-VIII) - Datta, K. B. (2000). <i>Matrix and Linear Algebra</i>. Prentice Hall of India Pvt. Ltd, New Delhi. - Shanti, N. (2012). <i>A Text Book of Matrices</i>. S. Chand Limited, New Delhi. - Bronson, R. (1989). <i>Schaum's Outline of Theory and Problems of matrices</i>. McGraw-Hill, New York. - John, V. (2010). <i>Mathematics for Computer Graphics</i>. Springer-Verlag London.
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List of Practicals

Practical No.-1: Adjoint of a Matrix

- Find the minors and cofactors of a matrix $A = \begin{bmatrix} -2 & 3 \\ 1 & -5 \end{bmatrix}$.
- For any square matrix A , prove that $A(adjA) = (adjA)A = |A|I$.
- If $A = \begin{bmatrix} 1 & 0 & 2 \\ -2 & 1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$, then with usual notations verify $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = |A|$.
- Find adjoint of $A = \begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$.
- If $A = \begin{bmatrix} -3 & 1 & 0 \\ 2 & -2 & 1 \\ -1 & -1 & 1 \end{bmatrix}$, then show that $A(adjA)$ is a null matrix.
- Let $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 1 & -1 & 1 \end{bmatrix}$. Show that $A(adjA)$ is an identity matrix.
- If A is a nonsingular matrix of order n , then show that $|adjA| = |A|^{n-1}$.
- If $A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$, then verify $A(adjA) = (adjA)A = |A|I$.
- If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ 5 & 9 \end{bmatrix}$, then verify that $adj(AB) = (adjB)(adjA)$.
- Let $A = \begin{bmatrix} 1 & -2 \\ 5 & -7 \end{bmatrix}$. Show that $adj(adjA) = A$.

Practical No.-2: Inverse of a Matrix

- Find the inverse of matrix $A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ by using adjoint method.
- Prove that a square matrix A has an inverse if and only if A is nonsingular.
- Verify whether the inverse of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ exists or not.
- Prove that an inverse of a matrix (if it exists) is unique.
- If A and B are nonsingular matrices of order n , then prove that $(AB)^{-1} = B^{-1}A^{-1}$.
- Find the inverse of matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ by using adjoint method.

- 7) If $A = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 2 & 1 \\ 3 & 4 & 1 \end{bmatrix}$, then verify that $\text{adj}(2A) = 2^2 \text{adj}A$.
- 8) If $A = \begin{bmatrix} 2 & 0 & -1 \\ 3 & 4 & 0 \\ -1 & 1 & 1 \end{bmatrix}$, then verify the result $(A^{-1})' = (A')^{-1}$.
- 9) Let $A = \begin{bmatrix} 1 & 2 & 4 \\ 7 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$, then find A^{-1} .
- 10) Find inverse of the matrix $A = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Practical No.-3: Elementary Matrices and Rank of Matrix

- 1) Compute the E-matrix E_{12} , $E_{12}(3)$ of order 3.
- 2) Compute the E-matrix $E_2(3) \cdot E_{12}(3)$ of order 3.
- 3) Write the E-matrix $[E_{21}]^{-1}$ and $[E_{21}(2)]^{-1}$
- 4) Compute the E-matrix $E'_2(3) \cdot E_{12}(2)$ of order 2.
- 5) Prove that inverse of an elementary matrix is an elementary matrix of the same type.
- 6) Find rank of the following matrix by using determinant method.
 - a. i) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ ii) $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$.
- 7) Find rank of the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 5 & 7 \\ 1 & 2 & 3 \end{bmatrix}$ by using determinant method.
- 8) Find rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ by using determinant method.
- 9) Prove that every nonsingular matrix can be expressed as a product of finite number of elementary matrices.
- 10) Express the nonsingular matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$ as a product of elementary matrices.

Practical No.-4: Normal Form and Rank of Matrix

- 1) Reduced the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 6 & 8 & 10 \end{bmatrix}$ to the normal form. Hence find it's rank.
- 2) Reduced the matrix $A = \begin{bmatrix} 3 & 2 & 5 & 7 & 12 \\ 1 & 1 & 2 & 3 & 5 \\ 3 & 3 & 6 & 9 & 15 \end{bmatrix}$ to the normal form. Hence find it's rank.
- 3) Reduced the matrix $A = \begin{bmatrix} 1 & 2 & 32 \\ 1 & 2 & 34 \\ 2 & 6 & 75 \end{bmatrix}$ to the normal form. Hence find it's rank.
- 4) Find the nonsingular matrices P and Q such that PAQ is in normal form, where $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$.

- 5) Find the nonsingular matrices P and Q such that PAQ is in normal form, where $A = \begin{bmatrix} 2 & 6 \\ 1 & 3 \\ 3 & 9 \end{bmatrix}$.
- 6) If $A = \begin{bmatrix} 2 & 0 & 2 \\ 0 & x & 1 \\ -2 & -1 & x \end{bmatrix}$ then show that $\rho(A) = 3, \forall x \in \mathbb{R} - \{-1\}$.
- 7) Determine the values of x , so that the matrix $A = \begin{bmatrix} x & x & 1 \\ 1 & x & x \\ x & 1 & x \end{bmatrix}$ is of rank 3.
- 8) Determine the value of x , so that the matrix $A = \begin{bmatrix} x & x & 1 \\ 1 & x & x \\ x & 1 & x \end{bmatrix}$ is of rank 2.
- 9) Determine the value of x , so that the matrix $A = \begin{bmatrix} x & x & 1 \\ 1 & x & x \\ x & 1 & x \end{bmatrix}$ is of rank 1.
- 10) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \\ 2 & 7 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 4 & 2 \\ 1 & 3 & 3 \end{bmatrix}$, then verify that $\rho(AB) \leq \min\{\rho(A), \rho(B)\}$.

Practical No.-5: System of Linear Equations-I

- Examine for consistency the following system of equations: $2x - 3y + 7z = 5$, $3x + y - 3z = 13$, $2x + 19y - 47z = 32$.
- Examine for consistency the following system of equations: $x + z = 2$, $-2x + y + 3z = 3$, $-3x + 2y + 7z = 4$.
- Examine the following system of equations for consistency and if consistent then solve them: $x + y + z = 6$, $2x + y + 3z = 13$, $5x + 2y + z = 12$, $2x - 3y - 2z = -10$.
- Test the following system of equations for consistency and solve them when consistent: $2x - y + 3z = 1$, $3x + 2y + z = 3$, $x - 4y + 5z = -1$.
- Test the following system of equations for consistency and solve them when consistent: $x + y + z = 6$, $x - y + 2z = 5$, $3x + y + z = 8$.
- Test the following system of equations for consistency and solve them when consistent: $-3x + 5z = 2$, $5x + y + 2z = 3$, $2x + y + 7z = -2$.
- Test the following system of equations for consistency and solve them when consistent: $x_1 + x_2 - 5x_3 - x_4 = 1$, $3x_1 - x_2 - 3x_3 - 11x_4 = -17$, $4x_1 + 5x_2 - 23x_3 - 2x_4 = 9$, $-x_1 + x_2 - x_3 + 5x_4 = 9$.
- If $A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & -2 & 3 \\ -1 & 3 & -4 \end{bmatrix}$, find A^{-1} . Hence solve the following system of linear equations: $2x + y - z = -1$, $x - 2y + 3z = 9$, $-x + 3y - 4z = -12$.
- If $A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$, find A^{-1} . Hence solve the system: $3x + y + 2z = 3$, $2x - 3y - z = -3$, $x + 2y + z = 4$.

- 10) Test the following equations for consistency and if consistent solve them: $2x - y - 5z + 4w = 1$, $x + 3y + z - 5w = 18$, $3x - 2y - 8z + 7w = -1$.

Practical No.-6: System of Linear Equations-II

- 1) Solve the following system of equations: $x_1 + 3x_2 + 4x_3 - 6x_4 = 0$, $x_2 + 6x_3 = 0$, $2x_1 + 2x_2 + 2x_3 - 3x_4 = 0$, $x_1 + x_2 - 4x_3 - 4x_4 = 0$.
- 2) Examine for non-trivial solutions: $x + y + z = 0$, $4x + y = 0$, $2x + 2y + 3z = 0$.
- 3) Examine for non-trivial solutions the following homogeneous system of linear equations: $x + y + 3z = 0$, $x - y + z = 0$, $-x + 2y = 0$, $x - y + z = 0$.
- 4) Examine for non-trivial solutions the following homogeneous system of linear equations: $2x + y + 3z + 6w = 0$, $3x - y + z + 3w = 0$, $-x - 2y + 3z = 0$, $-x - 4y - 2z - 7w = 0$.
- 5) For what value of a the equations: $x + y + z = 1$, $2x + 3y + z = a$, $4x + 9y - z = a^2$. Have a solution and solve them completely in each case.
- 6) Investigate for what values of λ and μ the following system of equations: $x + 3y + 2z = 2$, $2x + 7y - 3z = -11$, $x + y + \lambda z = \mu$ have (i) No solution (ii) a unique solution (iii) an infinite number of solutions.
- 7) Investigate for what values of λ and μ so that equations: $2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$, $2x + 3y + \lambda z = \mu$ have (i) No solution (ii) a unique solution (iii) an infinite number of solutions.
- 8) Investigate for what values of λ and μ so that equations: $2x - y + 3z = 2$, $x + y + 2z = 2$, $5x - y + \lambda z = \mu$ have (i) No solution (ii) a unique solution (iii) an infinite number of solutions.
- 9) Find the value of λ for which the following systems have a non-trivial solutions: $x + 2y + 3z = 0$, $2x + 3y + 4z = 0$, $3x + 4y + \lambda z = 0$.
- 10) Find the value of λ for which the following systems have a non-trivial solutions: $x + 2y + 3z = \lambda x$, $3x + y + 2z = \lambda y$, $2x + 3y + z = \lambda z$.

Practical No.-7: Eigen Vaules

- 1) Find the characteristics equation of $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$.
- 2) Find the characteristics equation of $A = \begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix}$.
- 3) Find the characteristics equation and eigen values of $A = \begin{bmatrix} 9 & -7 \\ 3 & -1 \end{bmatrix}$.
- 4) Find the eigen values of $A = \begin{bmatrix} 1 & -3 \\ -1 & 2 \end{bmatrix}$.
- 5) Find the eigen values of $A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.
- 6) Using eigen values of the matrix $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$, find the trace of A .

- 7) Using eigen values of the matrix $A = \begin{bmatrix} 9 & 8 \\ -6 & -5 \end{bmatrix}$, find the determinant of A .
- 8) If λ is a non-zero eigen value of a non-singular matrix A , then show that $\frac{1}{\lambda}$ is an eigen value of A^{-1} .
- 9) Find the eigen values of A^{-1} , where $A = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$.
- 10) Let k be a non-zero scalar and A be a non-zero square matrix. If λ is an eigen value of A , then show that $k\lambda$ is an eigen value of kA .

Practical No.-8: Cayley-Hamilton Theorem and Eigen Vectors

- 1) Find the eigenvalues of $\frac{1}{2}A$, where $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$.
- 2) Find the sum & product of the eigenvalues of the matrix $A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 3 & 1 \\ 2 & 1 & -6 \end{bmatrix}$.
- 3) The product of two eigenvalues of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16. Find the third eigenvalue of A .
- 4) Find all eigen values of $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 3 \end{bmatrix}$.
- 5) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $f(x) = -2 - 5x + x^2$. Show that $f(A)$ is a zero matrix of order 2.
- 6) Verify Cayley Hamilton theorem for $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$.
- 7) Verify Cayley Hamilton theorem for $A = \begin{bmatrix} 1 & -5 \\ 3 & 2 \end{bmatrix}$.
- 8) Find A^{-1} by using Cayley Hamilton theorem, where $A = \begin{bmatrix} 1 & -5 \\ 3 & 2 \end{bmatrix}$.
- 9) Find the eigen values and eigen vectors of $A = \begin{bmatrix} 9 & -7 \\ 3 & -1 \end{bmatrix}$.
- 10) Find the eigen values and eigen vectors of $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 6 \end{bmatrix}$.

F.Y. B.Sc. Mathematics (Open Elective)
Semester-I
MTH-OE-111: Mathematics for Competitive Exams-I

Total Hours: 30

Credits: 2

Course Objectives	- The main aim of introducing Mathematics for Competitive Examinations-I for students is to develop the skill to meet the competitive examinations for better job opportunities. - Effort has been made to accommodate fundamental, mathematical aspects to instill confidence among students. - Enrich their knowledge and to develop their logical reasoning thinking abilities. - Attract the students to attain the mathematical problems and create interest in students about mathematics.	
Course Outcomes	After successful completion of this course, students are expected to: - Have a strong base in the fundamental mathematical concepts. - Grasp the approaches and strategies to solve problems with speed and accuracy. - Gain appropriate skills to succeed in preliminary selection process for recruitment. - Solve problems easily by using simple method.	
Unit	Content	Hours
Unit I	Numbers: - Number System - Types of Numbers - series (AP and GP) - Algebraic operations BODMAS	7
Unit II	Divisibility: - Divisibility - LCM and HCF - Fraction and Simplification	8
Unit III	Time and Distance: - Time and Distance - Problems based on Trains - Boats and Streams	7
Unit IV	Clock and Calendar: - Time and work - Pipes and Cistern - Problems on Clock - Problems on Calendar	8

Study Resources	• Aggarwal, R.S. (2018). <i>Quantitative Aptitude for Competitive Examinations</i> (Revised ed.). S. Chand and Co. Ltd, New Delhi. (Section-I : Art.1-3, Art.7, Art.16-20, Art.27-28.) • Praveen, R. V. <i>Quantitative Aptitude and Reasoning</i>. PHI publishers. • <i>Quantitative Aptitude: Numerical Ability</i> (Fully Solved) Objective Questions. Kiran Prakashan, Pratogitaprakasan, Kic X, Kiran Prakasan publishers. • Guha, A. <i>Quantitative Aptitude for Competitive Examination</i>. Tata Mc Graw hill publications.	
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SEMESTER-II

F.Y. B.Sc. Mathematics (Major)
Semester-II
MTH-DSC-121: Theory of Equations

Total Hours: 30

Credits: 2

Course Objectives	- To study Principle of Mathematical Induction and Divisibility of numbers. - To study roots of polynomial equations and Fundamental theorem of algebra. - To know relations between roots and coefficients of polynomials of degree ≤ 4. - To know roots of cubic equations by using Cardon's method, biquadratic equations by Descarte's method and roots of polynomial equations by Newton's method.	
Course Outcomes	After successful completion of this course, students are expected to: - Use of Principle of Mathematical Induction and understand Divisibility of numbers with their properties. - Find out roots of any equation of degree less than or equal to five. - Know the relation between roots and coefficient of quadratic, cubic and biquadratic equations and their use for finding the roots of equation. - Use of Cardon's method, Descarte's method for solving equations.	
Unit	Content	Hours
Unit I	Divisibility of Integers: - Natural numbers - Well ordering principal (statement only) - Principle of Mathematical Induction - Divisibility of integers - Division algorithm - GCD and LCM - Euclidean algorithm - Unique factorization theorem	7
Unit II	Polynomials: - Revision of Polynomials - Horner's method of synthetic division - Existence and uniqueness of GCD of two polynomials - Polynomial equations - Factor theorem and generalized factor theorem for polynomials - Fundamental theorem of algebra (Statement only) - Methods to find common roots of polynomial equation - Descarte's rule of signs - Newton's method of divisors for the integral roots	8
Unit III	Theory of Equations-I: - Relation between roots and coefficient of general polynomial equation in one variable - Relation between roots and coefficient of quadratic - cubic and biquadratic equations - Symmetric functions of roots	7

Unit IV	Theory of Equations –II: • Transformation of equations • Cardon’s method of solving cubic equations • Biquadratic equations • Descarte’s method of solving biquadratic equations	8
Study Resources	• Sharma, D. R. (1985). <i>Theory of Equations</i>. Sharma Publications, Jalandhar. (Unit: I-VI) • Burton, D. M. (1989). <i>Elementary Number Theory</i>. W. C. Brown publishers, Dubuquolowa. (Unit-II: Art.2.1-2.4) • Hall, H. S., and Knight, S. R.(1994). <i>Higher Algebra</i>. H. M. Publications. • Datta,K. B. (2000). <i>Matrix and Linear Algebra</i>. Prentice Hall of India Pvt. Ltd., New Delhi.	

F.Y. B.Sc. Mathematics (Major)
Semester-II
MTH-DSC-122: Practical Course on Graph Theory

Total Hours: 60

Credits: 2

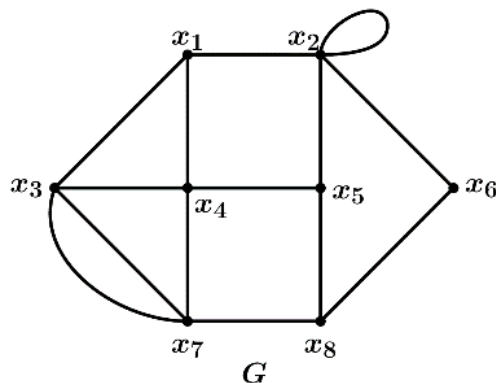
Course objectives	- To acquire concepts of graphs, connected graphs. - To know scope and importance of trees and directed graphs. - To know applications of the graphs.
Course outcomes	Upon successful completion of this course the student will be able to: - Understand the uses of the graph theory, designing issues in different problems like Konigsberg Seven Bridge Problem, Travelling salesman Problem. - Understand the issues associated with different types of graphs viz connected graphs, disconnected graphs. - Find the minimal Spanning trees. - Form flowchart using rooted trees.
Practical No.	Content
1	Graphs-I - Graph - Simple graph and Multigraph - Hand shaking lemma - Types of graphs
2	Graphs-II - Operations on graphs - Subgraphs - Isomorphism of graphs - Walk, Path, Cycles
3	Connected Graphs-I - Connected and disconnected graphs, - Bridges - Cut vertices - Edge connectivity and Vertex connectivity
4	Connected Graphs-II - Eulerian graph - Hamiltonian graph
5	Planner Graphs - Planer graph - Euler's formula for planer graphs - Kuratowski's second graph
6	Geometric Dual, Colouring of Graphs and Digraphs - Geometrical dual - Colouring of graphs - Digraphs
7	Trees-I - Trees - Distance and centre in a tree - radius, diameter and centre of graph

8	Trees-II • Rooted and Binary trees • Spanning trees • Minimal spanning trees
Study Resources	• Deo Narsingh (1979). <i>Graph Theory with Applications to Engineering and Computer science</i>. Prentice Hall of India Pvt. Ltd. (Ch.1: Sec.1.1 to 1.5, Ch.2: Sec. 2.1-2.2, 2.4-2.10, Ch.3: Sec. 3.1-3.8, 3.10, Ch.4 : Sec. 4.1-4.5) • Lipschitz Seymour and Lipson Marc Lars (2007). <i>Theory and Problems of Discrete Mathematics</i>. Schaum's outline series, McGraw-Hill Ltd., New York. • Clark, John. and Holton, Derek Allan. (1991). <i>A First Look At Graph Theory</i>. World Scientific. (Unit-VII: Sec.7.1-7.2) • Liu C. L. (1986). <i>Elements of Discrete Mathematics</i>. Mc Graw Hill, International Edition, Computer science series; Second Edition. • Harary F. (1969). <i>Graph Theory</i>. Addison-Wesley Publishing Company. • Bhav N. S. and Raghunathan T. T. (1990). <i>Elements of Graph Theory</i>. Goal Publications.

List of Practicals

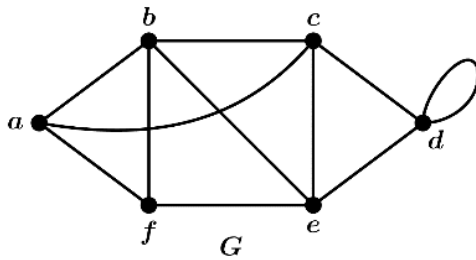
Practical No. 1: Graphs-I

- 1) Draw the graph $G = (V(G), E(G))$ if
 - a) $V(G) = \{v_1, v_2, v_3, v_4, v_5\}$ and $E(G) = \{(v_1, v_2), (v_1, v_3), (v_2, v_4), (v_3, v_4), (v_4, v_5)\}$,
 - b) $V(G) = \{a, b, c, d, e, f\}$ and $E(G) = \{(a, b), (b, b), (c, d), (d, f), (e, f)\}$.
- 2) Let G be the following graph.

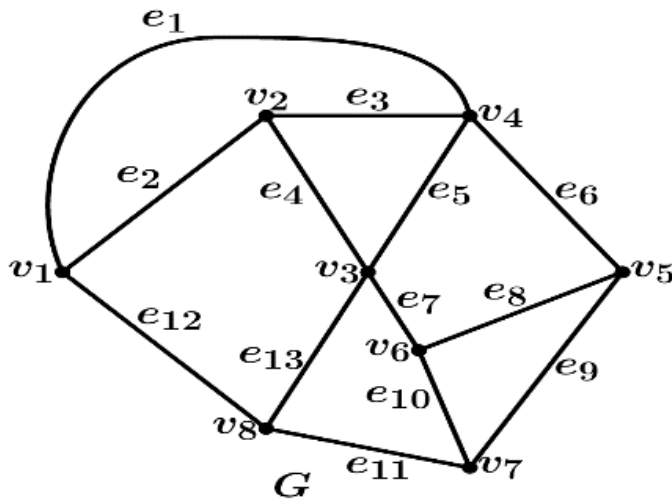


Find the degree of each vertex of graph G .

- 3) Prove that in any graph, the number of odd vertices is always even.
- 4) Prove that the maximum number of edges in a simple graph on n vertices is $\frac{n(n-1)}{2}$.
- 5) Show that the total number of edges in a complete graph on n vertices is $\frac{n(n-1)}{2}$.
- 6) Verify the Handshaking lemma for the following graph G .



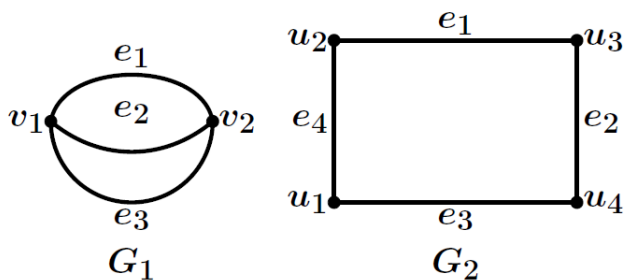
- 7) Find the subgraph $G - v_4$ and $G - \{e_7, e_8, e_{13}\}$ from the following graph G .



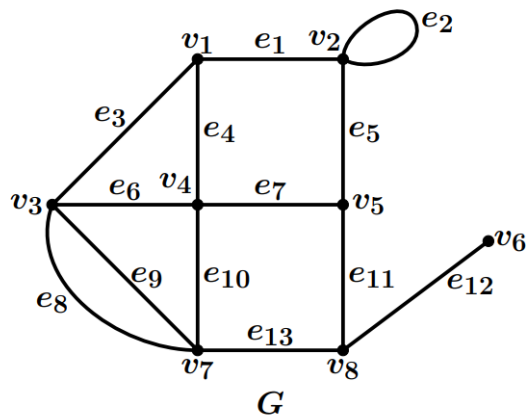
- 8) Let G be a simple regular graph with n vertices and 12 edges. Find all possible values of n and give example of G in any one case.
- 9) Does there exist a regular graph of degree 5 on 7 vertices? Justify.
- 10) Does there exist a graph on 5 vertices whose degree are 1, 2, 3, 4 and 5? Justify.

Practical No. 2: Graphs-II

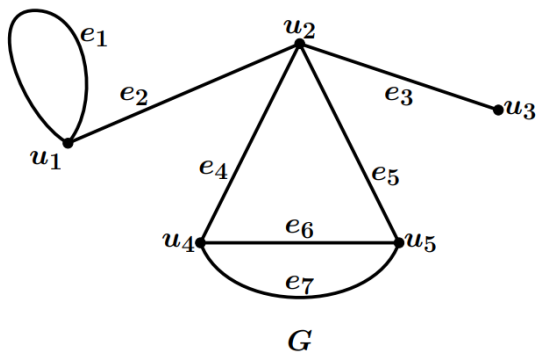
- 1) Find all subgraphs of the following graph G .



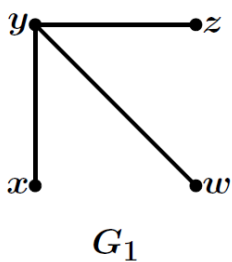
- 2) Let G be graph.



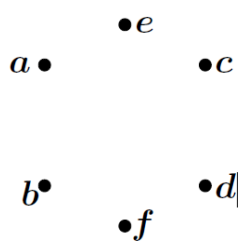
- I) Find the induced subgraph $G < U >$ where $U = \{u_2, u_3, u_4, u_7\}$.
- II) Find the induced subgraph $G < S >$ where $S = \{e_1, e_2, e_8, e_{11}\}$.
- 3) Find five spanning subgraphs of a graph G , given below.



- 4) Find the complement of the following graphs.

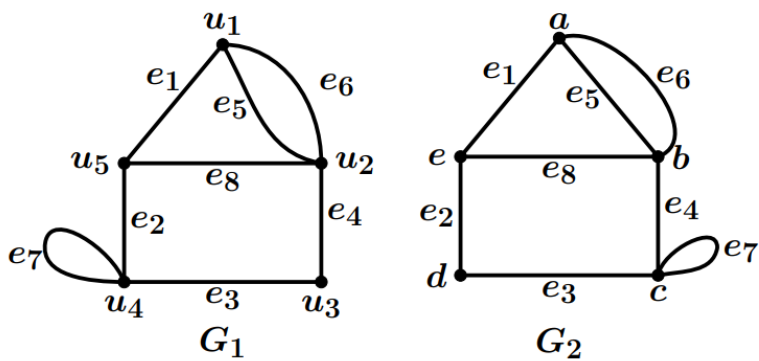


G_1

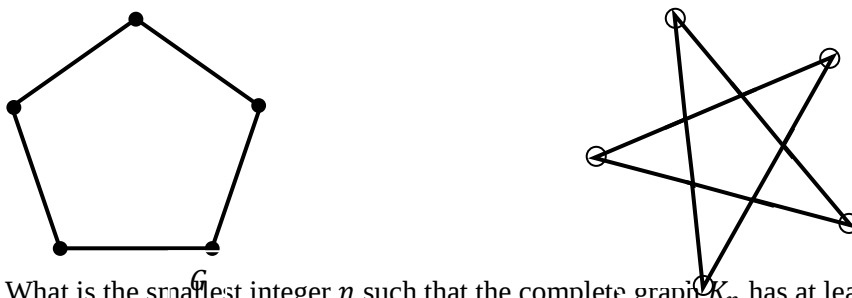


G_2

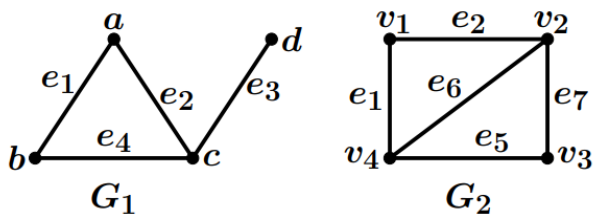
- 5) If G is self-complementary graph on n vertices, then show that n is of the type $4k$ or $4k + 1$ for some integer k .
- 6) Show that two graphs G_1 and G_2 given below are not isomorphic.



7) Show that the graphs G and G' given below are isomorphic.

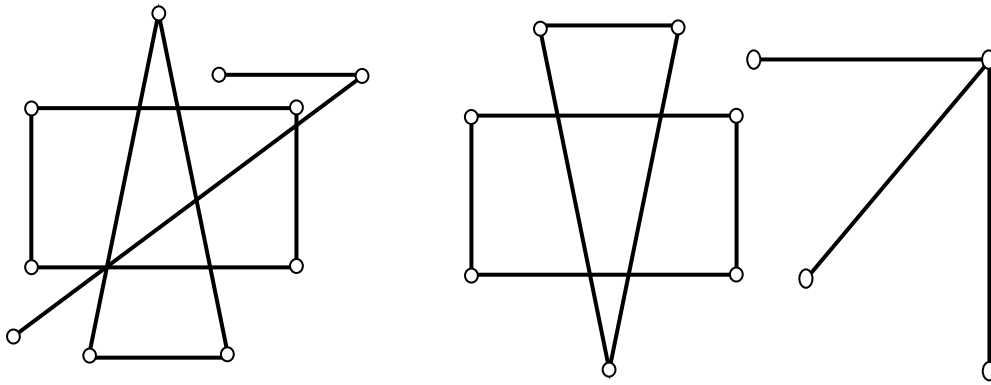


- 8) What is the smallest integer n such that the complete graph K_n has at least 300 edges?
- 9) Determine the number of edges in a graph with 6 vertices, 2 of degree 4 and 4 of degree 3. Draw two such graphs.
- 10) For the following graphs G_1 and G_2 . Find the graphs a) $G_1 \cup G_2$, b) $G_1 \cap G_2$, c) $G_1 \oplus G_2$.

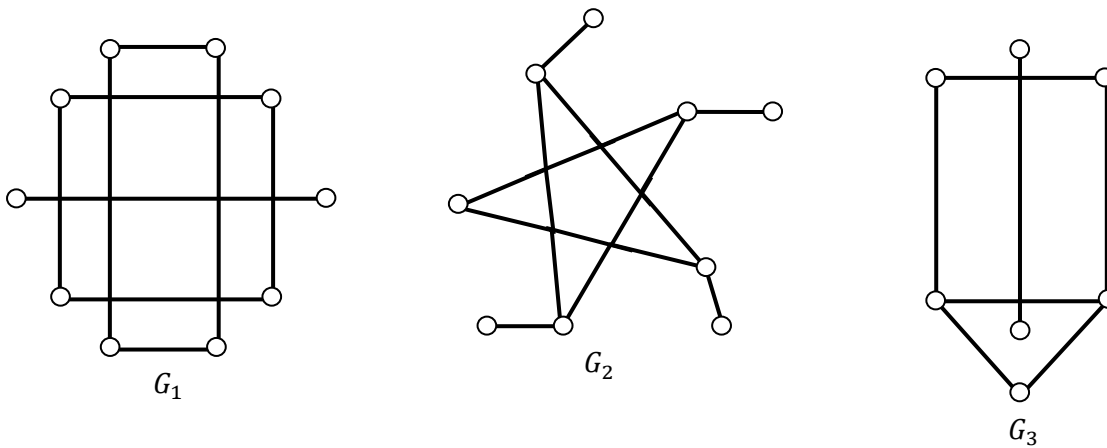


Practical No. 3: Connected Graphs-I

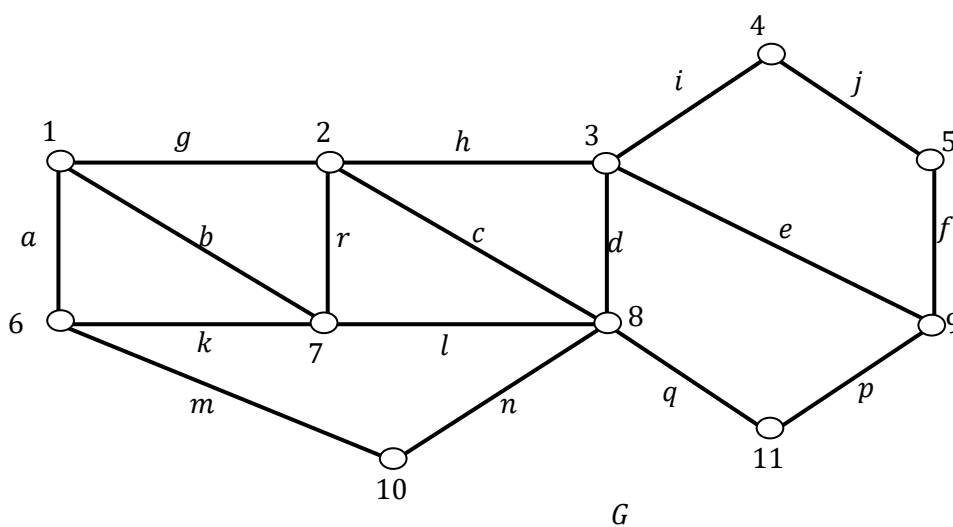
- 1) Which of the following graphs are connected? If not, then find components of the graph.



2) Are the following graphs connected? If it is disconnected find component of that graph.

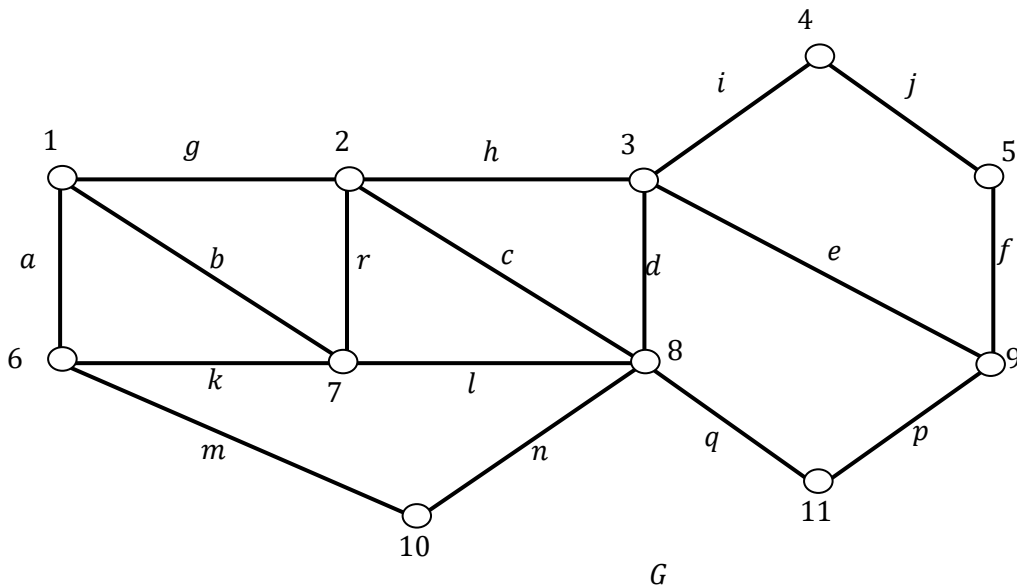


3) Consider the following graph G

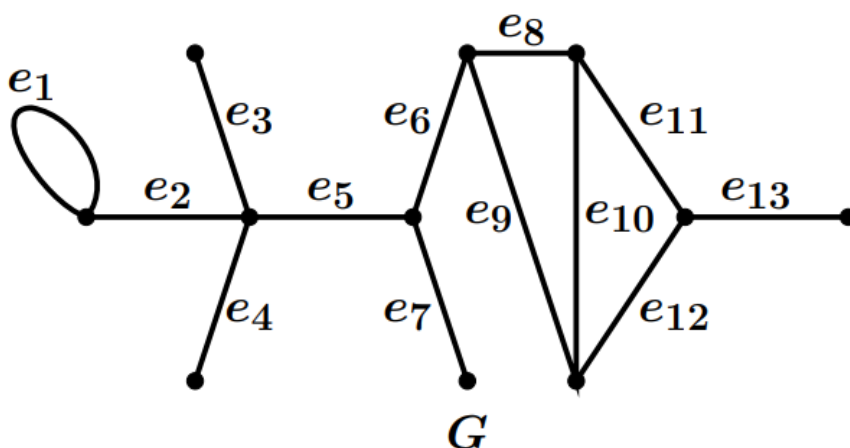


- Find a closed walk of length 8 in graph G .
- Find a path of length 8 in graph G .

- c) Find all edge disjoint paths between the vertices 1 and 8 in graph G .
- 4) Consider the following graph G



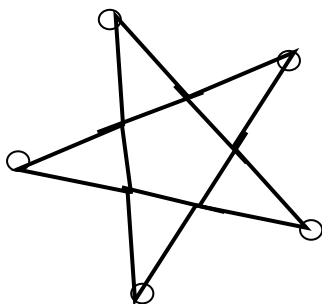
- a) Find the distance between vertices 6 and 9 in G .
- b) Find an open trail of length 8 in G .
- c) Find 5 different cut sets of G .
- d) Find $\lambda(G)$, $K(G)$ and $\delta(G)$.
- 5) If G is a simple graph with n vertices containing more than $\frac{(n-1)(n-2)}{2}$ edges, then show that G is connected.
- 6) Construct a graph G for which $K(G) = \lambda(G) = \delta(G)$.
- 7) Find all bridges in the following graph G .



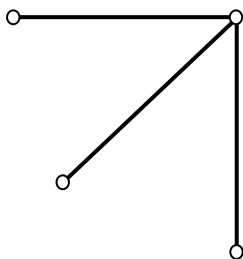
- 8) Construct a graph G for which $K(G) = 2$, $\lambda(G) = 3$ and $\delta(G) = 4$.
- 9) Find the edge connectivity of $K_{3,5}$, K_7 , K_6 .
- 10) Find the graph with 8 vertices and 16 edges whose edge connectivity is 4.

Practical No. 4: Connected Graphs-II

- 1) Draw a map and a graph representing the bridges of Königsberg.
- 2) Draw a graph with six vertices which is Eulerian but not Hamiltonian.
- 3) Draw a graph with six vertices which is Hamiltonian but not Eulerian.
- 4) Draw a graph with six vertices which is neither Hamiltonian nor Eulerian.
- 5) Draw a graph with six vertices which is both Hamiltonian and Eulerian.
- 6) Which of the following graphs are Eulerian ?
i) $K_{3,2}$ ii) $K_{2,4}$ iii) K_5
- 7) Which of the following graphs are Hamiltonian ?
i) $K_{3,2}$ ii) $K_{2,4}$ iii) K_5
- 8) Is the following graph G Eulerian? Justify.



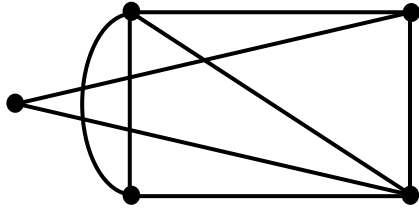
- 9) Is the following graph G Hamiltonian? Justify.



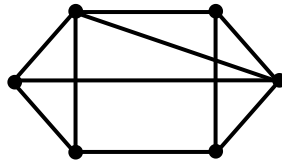
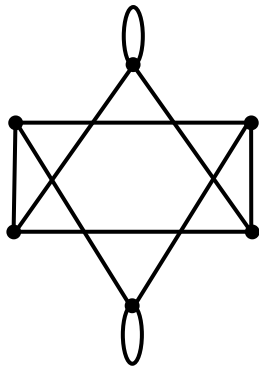
- 10) Verify the graph $K_{1,6}$ (star graph) for Eulerian and Hamiltonian.

Practical No. 5: Planner Graphs

- 1) Are the following graphs planar? Justify.
i) $K_{3,3}$ ii)



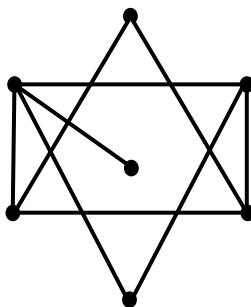
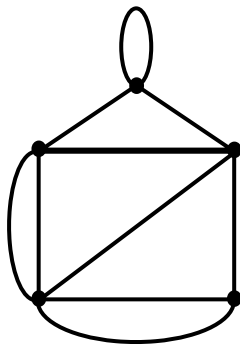
- 2) Show that Kuratowski's first graph is not a planar graph.
- 3) Show that Kuratowski's second graph is not a planar graph.
- 4) State and prove the Euler's formula for planar graph.
- 5) Verify Euler's formula for the following graphs.



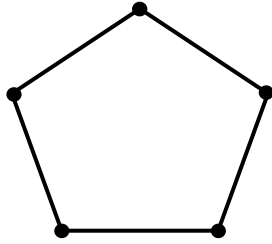
- 6) Find the number of edges in a simple graph with 16 vertices and 20 faces.
- 7) Find the number of faces in a simple graph with 7 vertices and 7 edges.
- 8) Let G be a 2-connected planar graph with p vertices and q edges. If $p \geq 3$, then $q \leq 3p - 6$.
- 9) Let G be a 2-connected graph. If G has p vertices and q edges, then $q \leq 2p - 4$.
- 10) Show that every tree is a planar graph.

Practical No. 6: Geometric Dual, Colouring of Graphs and Digraphs

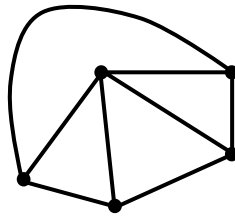
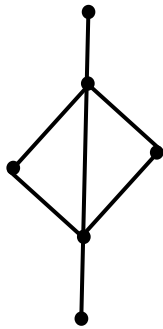
- 1) Construct geometric dual of the following graphs?



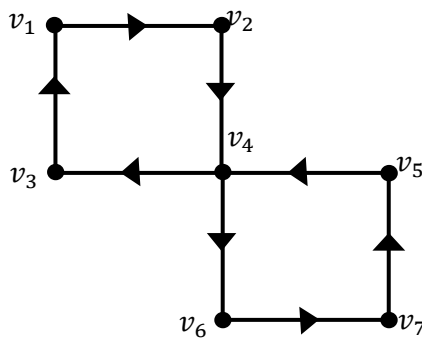
- 2) Show that the geometric dual of K_4 is K_4 .
- 3) Show that the following graph G is a self dual graph.



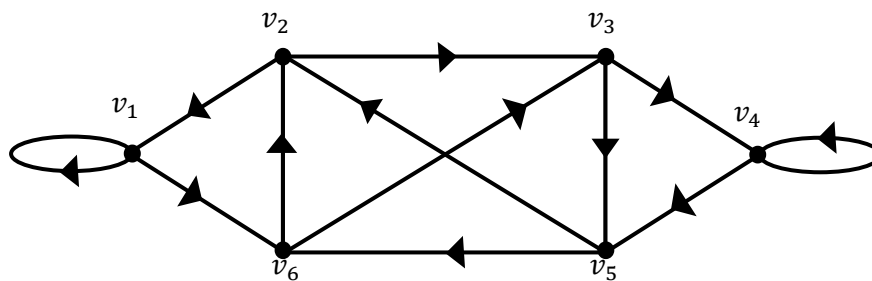
- 4) Prove that every tree having at least two vertices is 2-chromatic.
- 5) Find all graphs which are 1-colourable.
- 6) Determine the chromatic number of the following graphs.



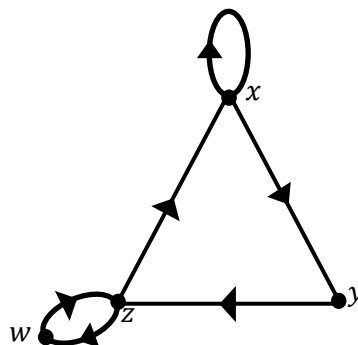
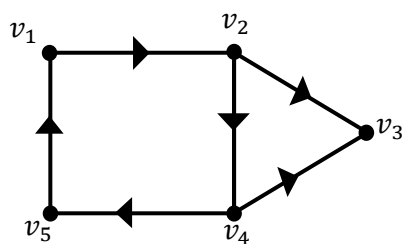
- 7) Find in degree and out degree of each vertex in the following digraph.



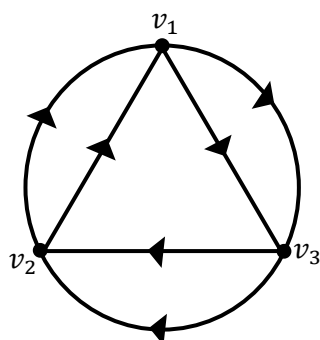
- 8) Verify $\sum d^+(v_i) = \sum d^-(v_i)$ for the following digraph.



- 9) Are the following digraphs balanced? Justify.

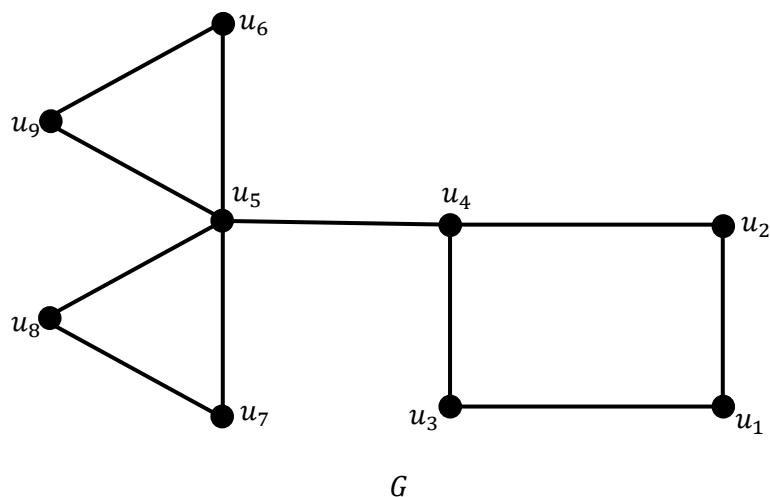


10) Verify the following digraph is regular or not?



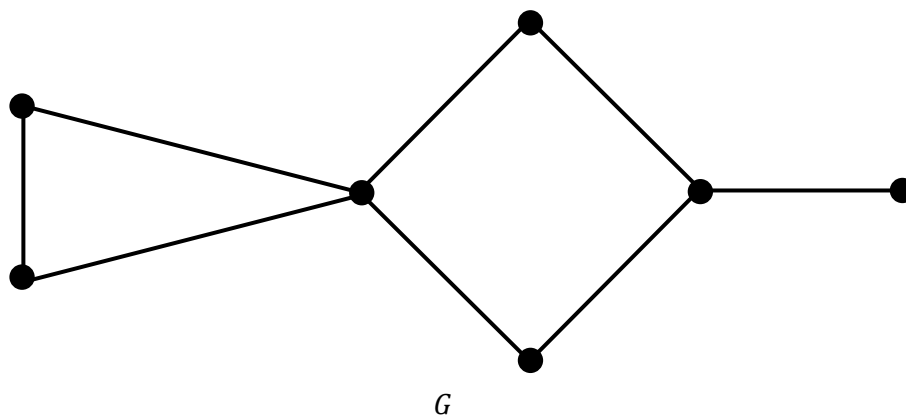
Practical No. 7: Trees-I

- 1) Draw all non isomorphic trees on 6 vertices.
- 2) Draw all non isomorphic trees on 7 vertices.
- 3) Construct a tree on 8 vertices having minimum number of pendant vertices.
- 4) Construct a tree on 8 vertices having maximum number of pendant vertices.
- 5) A tree has $2n$ pendant vertices, $3n$ vertices of degree 2 and n vertices of degree 3. Determine number of vertices and edges in a tree.
- 6) A tree has two vertices of degree 2, one vertex of degree 3 and three vertices of degree 4. How many number of vertices of degree 1 does it have?
- 7) Prove that there is a unique path between every pair of vertices of a tree.
- 8) Prove that a tree with n vertices must have $n - 1$ edges.
- 9) Prove that in a binary tree with n vertices: (i) The number of vertices is odd. (ii) $= \frac{n+1}{2}$, where p is the number of pendant vertices. (iii) $q = p - 1$, where q is the number of non pendant vertices.
- 10) Find the eccentricity of each vertex of the following graph G . Also find radius, diameter and centre of graph G .

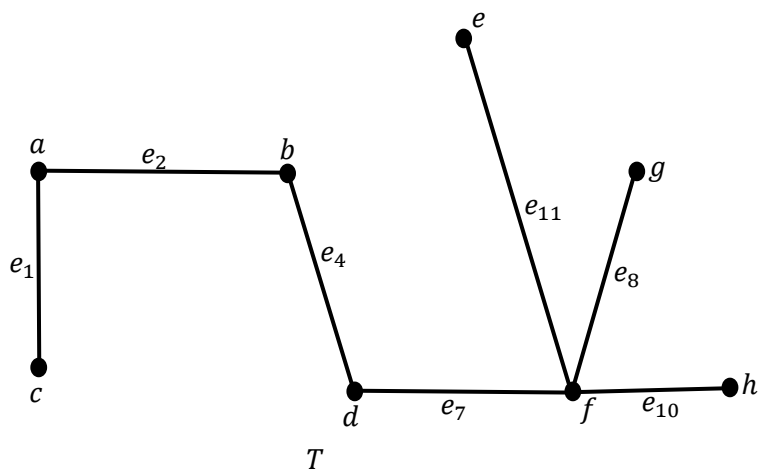
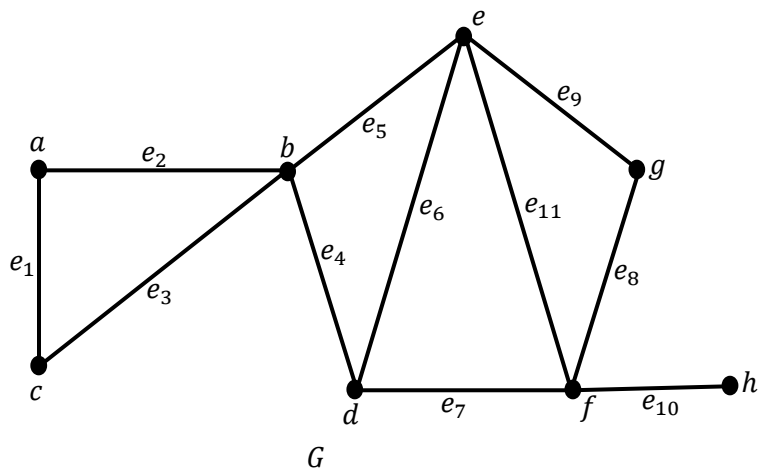


Practical No. 8: Trees-II

- 1) Construct a tree on 8 vertices in which its diameter is not equal to twice the radius of it.
- 2) Find all the rooted trees with 4 vertices.
- 3) Find all the spanning trees of the following graph G .



- 4) Draw all the spanning trees of the graph K_4 .
- 5) Find the rank and nullity of the following graphs (i) $K_{3,3}$ (ii) K_7 .
- 6) Find the fundamental cycles and fundamental cutsets of the following graph G with respect to spanning tree T .



- 7) Sketch the binary tree with 6 pendant vertices.
- 8) A binary tree T has 7 pendant vertices. Find the number of vertices of degree 3 in T .
- 9) Find a graph G in which each vertex is centre such that radius of graph G is same as diameter of graph G .
- 10) Is it possible to construct a tree with 6 vertices and having total degree 14? Explain.

F.Y. B.Sc. Mathematics (Open Elective)
Semester-II
MTH-OE-121: Mathematics for Competitive Exams-II

Total Hours: 30

Credits: 2

Course Objectives	- The main aim of introducing Mathematics for Competitive Examinations-II for students is to develop the skill to meet the Competitive examinations for better job opportunities. - Effort has been made to accommodate fundamental, mathematical aspects to instill confidence among students. - Enrich their knowledge and to develop their logical reasoning thinking abilities. - Attract the students to attain the mathematical problems and create interest in students about mathematics.	
Course Outcomes	After successful completion of this course, students are expected to: - Have a strong base in the fundamental mathematical concepts. - Grasp the approaches and strategies to solve problems with speed and accuracy. - Gain appropriate skills to succeed in preliminary selection process for recruitment. - Solve problems easily by using simple method.	
Unit	Content	Hours
Unit I	Percentage and Average: - Percentage and Average - Problems based on Ages - Ratio and Proportion	7
Unit II	Partnership and Share: - Partnership - Stocks and Shares	8
Unit III	Profit and Loss: - Profit and Loss - Simple Interest - Compound Interest	7
Unit IV	Permutations and Combinations: - Permutations and Combinations - Probability - True discount and Banker's discount	8
Study Resources	- Aggarwal, R.S. (2018). <i>Quantitative Aptitude for Competitive Examinations</i> (Revised ed.). S. Chand and Co. Ltd, New Delhi. (Section-I: Art.6, Art.8, Art.11-14, Art.22-23, Art.29-33.) - Praveen, R. V. <i>Quantitative Aptitude and Reasoning</i>. PHI publishers. <i>Quantitative Aptitude: Numerical Ability (Fully Solved)</i> Objective Questions. Kiran Prakashan, Pratogitaprakasan, Kic X, Kiran Prakashan publishers. - Guha, A. <i>Quantitative Aptitude for Competitive Examination</i>. Tata Mc Graw hill publications.	

F.Y. B.Sc. Mathematics (Open Elective)
Semester-II
MTH-OE-122: Practical Course on Mathematics for Competitive Exams

Total Hours: 60

Credits: 2

Course Objectives	- The main aim of introducing Mathematics for Competitive Examinations for students is to develop the skill to meet the Competitive examinations for better job opportunities. - Effort has been made to accommodate fundamental, mathematical aspects to instill confidence among students. - Enrich their knowledge and to develop their logical reasoning thinking abilities. - Attract the students to attain the mathematical problems and create interest in students about mathematics.
Course Outcomes	After successful completion of this course, students are expected to: - Have a strong base in the fundamental mathematical concepts. - Grasp the approaches and strategies to solve problems with speed and accuracy. - Gain appropriate skills to succeed in preliminary selection process for recruitment. - Solve problems easily by using simple method.

List of Practicals

Practical No. 1: Percentage

- Express each of the following as a decimal: 6%, 28%, 0.2%, 0.04%, $\frac{1}{2}\%$.
- Evaluate: (i) 70% of 320 + 45% of 240, (ii) $16\frac{2}{3}\%$ of 600 gm – $33\frac{1}{3}\%$ of 180 gm.
- Find the missing figures: (i) ?% of 25 = 2.125, (ii) 9% of ? = 63, (iii) 0.25% of ? = 0.04.
- Saroj invests Rs.72318/-, which is 17% of her annual income, in mutual funds. What is her monthly income?
- An inspector rejects 0.08% of the meters as defective. How many will he examine to reject 2?
- The difference between 31% of a number and 13% of the same number is 576. What is 17% of that number?
- Sixty-five percent of a number is 21 less than four-fifths of that number. What is the number?
- The difference between the value of a number increased by 25% and the value of the original number decreased by 30% is 22. What is the original number?
- Mrs. Roy spent Rs.44620/- on Deepawali shopping, Rs.32764/- on buying laptop and the remaining 32% of the total amount she had as cash with her. What was the total amount?
- The monthly income of a person was Rs.13500/- and his monthly expenditure was Rs.9000/-. Next year, his income increased by 14% and his expenditure by 7%. Find the percentage increase in his savings.

Practical No. 2: Average

- There are six numbers 30, 72, 53, 68, x and 87, out of which x is unknown. The average of the numbers is 60. What is the value of x ?
- Find the average of first 40 natural numbers.
- A man bought 5 shirts at Rs. 450/- each, 4 trousers at Rs. 750/- each and 12 pairs of shoes at Rs. 750/- each. What is the average expenditure per article?
- 13 chairs and 5 tables were bought for Rs. 8280/-. If the average cost of a table be Rs. 1227/-, what is the

average cost of a chair?

- 5) The average of five consecutive numbers A, B, C, D and E is 48. What is the product of A and E?
- 6) The average age of a man and his son is 40 years. The ratio of their ages is 11 : 5 respectively. What is the son's age?
- 7) The average of five numbers is 58. The average of the first two numbers is 48.5 and the average of last two numbers is 53.5. What is the third number?
- 8) The average age of A and B is 20 years. If A is replaced by C, the average age becomes 19 years and if B is replaced by C, the average age becomes 21 years. Find the ages of A, B and C.
- 9) The average age of a class was 15 years. When 5 boys whose average age was 12 years 6 months were admitted in the class, the average was reduced by 6 months. How many students were there in the class originally?
- 10) The average score of girls in class X examination in a school is 73 and that of boys is 71. The average score in class X of that school is 71.8. Find the percentage of the number of girls and boys in class X of the school.

Practical No. 3: Partnership

- 1) A, B and C started a business by investing Rs.120000/-, Rs.135000/- and Rs.150000/- respectively. Find the share of each, out of an annual profit of Rs.56700/-.
- 2) A, B and C start a business each investing Rs.20000/-. After 5 months A withdrew Rs.5000/-, B withdrew Rs.4000/- and C invests Rs.6000/- more. At the end of the year, a total profit of Rs.69900/- was recorded. Find the share of each.
- 3) A, B and C enter into partnership. A invests 3 times as much as B invests and B invests two-thirds of what C invests. At the end of the year, the profit earned is Rs.6600/-. What is the share of B?
- 4) Four milkmen rented a pasture. A grazed 24 cows for 3 months; B 10 cows for 5 months; C 35 cows for 4 months and D 21 cows for 3 months. If A's share of rent is Rs.720/-, find the total rent of the field.
- 5) A, B and C took a house on rent for one year for Rs.13824/-. They remained together for 4 months and then C left the house. After 5 more months, B also left the house. How much rent should each pay?
- 6) The ratio of investments of two partners A and B is 11 : 12 and the ratio of their profits is 2 : 3. If A invested the money for 8 months, then for how much time B invested his money?
- 7) A, B and C enter into a partnership by investing in the ratio of 3 : 2 : 4. After one year, B invests another Rs.270000/- and C, at the end of 2 years, also invests Rs.270000/-. At the end of three years, profits are shared in the ratio of 3 : 4 : 5. Find the initial investment of each.
- 8) A, B and C enter into a partnership. Their capital contribution is in the ratio 21 : 18 : 14. At the end of the business term they share profits in the ratio 15 : 8 : 9. Find the ratio of time for which they invest their capitals.
- 9) A invested Rs.76000/- in a business. After few months, B joined him with Rs.57000/-. At the end of the year, the total profit was divided between them in the ratio 2 : 1. After how many months did B join?
- 10) Alfred started a business investing Rs.45000/-. After 3 months, Peter joined him with a capital of Rs.60000/-. After another 6 months, Ronald joined them with a capital of Rs.90000/-. At the end of the year, they made a profit of Rs.16500/-. Find the share of each.

Practical No. 4: Stocks and Shares

- 1) Find the annual income derived from Rs.2500/-, 8% stock at 106.
- 2) Find the annual income derived by investing Rs.6800/- in 10% stock at 136.
- 3) Which is better investment? $7\frac{1}{2}$ % stock at 105 or $6\frac{1}{2}$ % stock at 94.
- 4) Find the cost of 96 shares of Rs.10/- each at $\frac{3}{4}$ discount, brokerage being $\frac{1}{4}$ per share.

- 5) Chinmay invested 25%, 30% and 20% of his savings in buying shares of three different companies P, Q and R which declared dividends 10%, 12% and 15% respectively. If his total income on account of dividends be Rs.5460/-, find the amount he invested in buying shares of the company Q.
- 6) Find the income derived from 88 shares of Rs.25/- each at 5 premium, brokerage being $\frac{1}{4}$ per share and the rate of dividend being $7\frac{1}{2}$ % per annum. Also, find the rate of interest on the investment.
- 7) Ravi invested Rs.913/- partly in 4% stock at Rs.97/- and partly in 5% stock at Rs.107/-. If his income from both is equal, find the amount invested in each stock.
- 8) A man buys Rs.25/- shares in a company which pays 9% dividend. The money invested is such that it gives 10% on investment. At what price did he buy the shares?
- 9) A man sells Rs.5000/-, 12% stock at 156 and invests the proceeds partly in 8% stock at 90 and 9% stock at 108. He thereby increases his income by Rs.70/-. How much of the proceeds were invested in each stock?
- 10) A man invested Rs.14,400/- in Rs.100/- shares of a company at 20% premium. If the company declares 5% dividend at the end of the year, then how much does he get?

Practical No. 5: Profit and Loss

- 1) Mansi purchased a car for Rs.250000/- and sold it for Rs.348000/-. What is the percent profit she made on the car?
- 2) Shaloo sold a mobile phone at the cost of Rs. 1950/- at a loss of 25%. At what cost will she have to sell it to get a profit of 30%?
- 3) A television manufacturer earns 20% profit by selling each T.V. set for Rs.14400/-. If the production cost is increased by 15%, what should be the new selling price of a set so as to gain 15%?
- 4) A book was sold for Rs. 27.50/- with a profit of 10%. If it were sold for Rs. 25.75/-, then what would have been the percentage of profit or loss?
- 5) A manufacturer makes 800 articles at a cost of Rs. 1.50/- per article. He fixes the selling price such that if only 600 articles are sold, he would make a profit of 30% on his outlay. However, he sold 620 articles at this price. Find his actual profit percent of the total outlay, assuming that the unsold articles are useless.
- 6) By selling 33 metres of cloth, one gains the selling price of 11 metres. Find the gain percent.
- 7) A vendor bought bananas at 6 for Rs. 10/- and sold them at 4 for Rs. 6/-. Find his gain or loss percent.
- 8) A grocer purchased 80 kg of sugar at Rs. 13.50/- per kg and mixed it with 120 kg sugar at Rs. 16/- per kg. At what rate should he sell the mixture to gain 16%?
- 9) A shopkeeper advertises for selling cloth at 4% loss. However, by using a false metre scale he actually gains 25%. What is the actual length of the scale?
- 10) Two-thirds of a consignment was sold at a profit of 5% and the remainder at a loss of 2%. If the total profit was Rs. 400/-, then find the value of the consignment.

Practical No. 6: Simple and Compound Interest

- 1) Find the simple interest on Rs. 68000/- at $16\frac{2}{3}$ % per annum for 9 months.
- 2) Find the simple interest on Rs. 3000/- at $6\frac{1}{4}$ % per annum for the period from 4th Feb., 2009 to 18th April, 2009.
- 3) A sum at simple interest at $13\frac{1}{2}$ % per annum amounts to Rs. 2502.50/- after 4 years. Find the sum.
- 4) The simple interest accrued on an amount of Rs. 2500/- at the end of 6 years is Rs. 1875/-. What would be the simple interest accrued on an amount of Rs. 6875/- at the same rate and for the same period?
- 5) A sum of Rs. 800/- amounts to Rs. 920/- in 3 years at simple interest. If the interest rate is increased by 3%, then it would amount to how much ?

- 6) After 3 years, how much compound interest will be obtained on Rs. 7800/- at the interest rate of 5% per annum?
- 7) Find the compound interest on Rs. 8000/- at 15% per annum for 2 years 4 months, compounded annually.
- 8) Find the compound interest on Rs. 10000/- in 2 years at 4% per annum, the interest being compounded half-yearly.
- 9) Find the compound interest on Rs. 16000/- at 20% per annum for 9 months, compounded quarterly.
- 10) The simple interest accrued on an amount of Rs. 40000/- at the end of four years is Rs. 24000/-. What would be the compound interest accrued on the same amount at the same rate in the same period?

Practical No. 7: Permutations and Combinations

- 1) Evaluate: i) 5P_5 ii) ${}^{50}P_3$.
- 2) Evaluate: i) ${}^{10}C_3$ ii) ${}^{100}C_{98}$ iii) ${}^{60}C_{60}$.
- 3) How many arrangements can be made out of the letters of the word 'ENGINEERING'?
- 4) In how many different ways can the letters of the word 'DESIGN' be arranged so that the vowels are at the two ends?
- 5) In how many different ways can the letters of the word 'DAUGHTER' be arranged so that the vowels always come together?
- 6) In how many different ways can the letters of the word 'DIGEST' be arranged so that the vowels are never together?
- 7) In how many different ways can the letters of the word 'DETAIL' be arranged so that the vowels occupy only the odd positions?
- 8) In how many ways can a cricket eleven be chosen out of 14 players?
- 9) In how many ways, a committee of 6 members be selected from 7 men and 5 ladies, consisting of 4 men and 2 ladies?
- 10) A committee of 5 members is to be formed by selecting out of 4 men and 5 women. In how many different ways the committee can be formed if it should have at least 1 man?

Practical No. 8: Probability, True Discount and Banker's Discount.

- 1) In a throw of a coin, find the probability of getting a head.
- 2) Two unbiased coins are tossed. What is the probability of getting at most one head?
- 3) An unbiased die is tossed. Find the probability of getting a multiple of 3.
- 4) In a simultaneous throw of a pair of dice, find the probability of getting a total more than 7.
- 5) A bag contains 6 white and 4 black balls. Two balls are drawn at random. Find the probability that they are of the same colour.
- 6) Two cards are drawn at random from a pack of 52 cards. What is the probability that either both are black or both are queens?
- 7) Find the present worth of Rs.930/- due 3 years hence at 8% per annum. Also find the discount.
- 8) The true discount on a bill due 9 months hence at 12% per annum is Rs.540/-. Find the amount of the bill and its present worth.
- 9) If the true discount on a certain sum due 6 months hence at 15% is Rs.120/-, then what is the banker's discount on the same sum for the same time and at the same rate?
- 10) The banker's discount on Rs.1800/- at 12% per annum is equal to the true discount on Rs.1872/- for the same time at the same rate. Find the time.