Date :- 25/04/2025

NOTIFICATION

Sub :- CBCS Syllabi of B. Sc. in Mathematics (Sem. V & VI)

Ref. :- Decision of the Academic Council at its meeting held on 22/04/2025.

The Syllabi of B. Sc. in Mathematics (Fifth and Sixth Semesters) as per **NATIONAL EDUCATION POLICY – 2020 (2023 Pattern)** and approved by the Academic Council as referred above are hereby notified for implementation with effect from the academic year 2025-26.

Copy of the Syllabi Shall be downloaded from the College Website (www.kcesmjcollege.in)

Sd/-
Chairman,
Board of Studies

To :

1) The Head of the Dept., M. J. College, Jalgaon.
2) The office of the COE, M. J. College, Jalgaon.
3) The office of the Registrar, M. J. College, Jalgaon.

Khandesh College Education Society's

Moolji Jaitha College, Jalgaon

An "Autonomous College"

Affiliated to

Kavayitri Bahinabai Chaudhari
North Maharashtra University, Jalgaon-425001

KNOWLEDGE IS POWER

ESTD. 1945

STRUCTURE AND SYLLABUS

B.Sc. Honours/Honours with Research (T.Y. B.Sc. Mathematics)

Under Choice Based Credit System (CBCS) and as per NEP-2020 Guidelines

[w.e.f. Academic Year: 2025-26]

Preface

The Moolji Jaitha College (Autonomous) has adopted a department-specific model as per the guidelines of UGC, NEP-2020 and the Government of Maharashtra. The Board of Studies in Mathematics of the college has prepared the syllabus for the third-year graduate of Mathematics. The syllabus cultivates theoretical knowledge and applications of different fields of Mathematics. The contents of the syllabus have been prepared to accommodate the fundamental aspects of various disciplines of Mathematics and to build the foundation for various applied sectors of Mathematics. The program will be enlightened the students with the advanced knowledge of Mathematics, which will help to enhance student's employability.

The overall curriculum of three/four year covers pure mathematics, applied mathematics and computational mathematics with programming. The syllabus is structured to cater the knowledge and skills required in the research field, Industrial Sector and Entrepreneurship etc.. The detailed syllabus of each paper is appended with a list of suggested readings.

Hence, Board of Studies in Mathematics in its meeting held on 22/03/2025 resolved to accept the revised syllabus for T. Y. B. Sc. (Mathematics) based on Choice Based Credit System (CBCS) of UGC, NEP-2020 and the Government of Maharashtra guidelines.

Program Outcomes (PO) for B.Sc. Program:

Program outcomes associated with a B.Sc. degree are as follows:

PO No.	PO
1	Graduates should have a comprehensive knowledge and understanding of the fundamental principles, theories and concepts in their chosen field of study.
2	Graduates should possess the necessary technical skills and competencies related to their discipline, including laboratory techniques and data analysis.
3	Graduates should be able to identify, analyze and solve complex problems using logical and critical thinking skills. They should be able to apply scientific methods and principles to investigate and find solutions of problem.
4	Graduates should be proficient in effectively communicating scientific information, both orally and in writing.
5	Graduates should have a basic foundation in research methods and be capable of designing and conducting scientific investigations.
6	Graduates should be able to work effectively as part of a team, demonstrating the ability to collaborate with others, respect diverse perspectives and contribute to group projects.
7	Graduates should recognize the importance of ongoing learning and professional development. They should be equipped with the skills and motivation to engage in continuous learning, adapt to new technologies and advancements in their field and stay updated with current research.

Programme Specific Outcome (PSO) for B.Sc. (Mathematics) Honours/Honours with Research:

After completion of this program, students are expected to learn/understand the:

PSO No.	PSO
1	Demonstrate the concepts involved in Real Analysis, Matrix Theory, Differential Equations, Algebra, Number Theory and Applied Mathematics.
2	Gain proficiency in mathematical techniques of both pure and applied mathematics and will be able to apply necessary mathematical methods to a scientific problem.
3	Acquire significant knowledge on various aspects related to Linear Algebra, Metric Spaces,

	Lattice Theory, Integral Transforms, Optimization Techniques and Partial Differential Equations.
4	Learn to work independently as well as a team to formulate appropriate mathematical methods.
5	Develope the ability to understand and use the morality and ethics related to scientific research.
6	Realize the scope of Mathematics and plan to continue their education as a Post-Graduate student of Mathematics and contribute to Mathematics through their research as a researcher.

Multiple Entry and Multiple Exit options:

The multiple entry and exit options with the award of UG certificate/ UG diploma/ or three-year degree depending upon the number of credits secured;

Levels	**Qualification Title**	**Credit Requirements**		**Semester**	**Year**
		Minimum	**Maximum**		
4.5	UG Certificate	40	44	2	1
5.0	UG Diploma	80	88	4	2
5.5	Three Year Bachelor's Degree	120	132	6	3
6.0	Bachelor's Degree- Honours **Or** Bachelor's Degree- Honours with Research	160	176	8	4

Credit distribution structure for Three/ Four year Honors/ Honors with Research Degree Programme with Multiple Entry and Exit

F.Y. B.Sc.

Year (Level)	**Sem**	**Subject-I (M-1)**	**Subject-II (M-2)**	**Subject-III (M-3)**	**Open Elective (OE)**	**VSC, SEC (VSEC)**	**AEC, VEC, IKS**	**CC, FP, CEP, OJT, RP**	**Cumulative Credits/Sem**	**Degree/ Cumulative Credit**
I (4.5)	I	DSC-1(2T) DSC-2(2P)	DSC-1(2T) DSC-2(2P)	DSC-1(2T) DSC-2(2P)	OE-1(2T)	---	AEC-1(2T) (Eng) VEC-1(2T) (ES) IKS(2T)	CC-1(2T)	22	**UG Certificate**
	II	DSC-3(2T) DSC-4(2P)	DSC-3(2T) DSC-4(2P)	DSC-3(2T) DSC-4(2P)	OE-2(2T) OE-3(2P)	---	AEC-2(2T) (Eng) VEC-2(2T) (CI)	CC-2(2T)	22	
	Cum. Cr.	**8**	**8**	**8**	**6**	**---**	**10**	**4**	**44**	
Exit option: Award of UG Certificate with 44 credits and an additional 4 credits core NSQF course/ Internship OR Continue with Major and Minor.										

S.Y. B.Sc.

Year (Level)	**Sem**	**Subject-I (M-1) Major***		**Subject-II (M-2) Minor #**	**Subject-III (M-3)**	**Open Elective (OE)**	**VSC, SEC (VSEC)**	**AEC, VEC, IKS**	**CC, FP, CEP, OJT/Int/RP**	**Cumulative Credits/Sem**	**Degree/ Cumulative Credit**
		Mandatory (DSC)	**Elective (DSE)**	**(MIN)**							
2 (5.0)	III	DSC-5(2T) DSC-6(2T) DSC-7(2P)	---	MIN-1(2T) MIN-2(2T) MIN-3(2P)	---	OE-4(2T)	SEC-1(2T)	AEC-3(2T) (MIL)	CC-3(2T) CEP(2)	22	**UG Diploma**
	IV	DSC-8(2T) DSC-9(2T) DSC-10(2P)	---	MIN-4(2T) MIN-5(2P)	---	OE-5(2T)	SEC-2(2T) SEC-3(2P)	AEC-4(2T) (MIL)	CC-4(2T) FP(2)	22	
	Cum . Cr.	**12**	**---**	**10**	**---**	**4**	**6**	**4**	**8**	**44**	
	Exit option: Award of UG Diploma in Major and Minor with 88 credits and an additional 4 credits core NSQF course/ Internship OR Continue with Major & Minor.										

***** Student must choose one subject as a Major subject out of M-1, M-2 and M-3 that he/she has chosen at First year

Student must choose one subject as a Minor subject out of M-1, M-2 and M-3 that he/she has chosen at First year (Minor must be other than Major)

۞ OJT/Internship/CEP should be completed in the summer vacation after 4th semester

T.Y. B.Sc.

Year (Level)	Sem	Subject-I (M-1) Major		Subject-II (M-2) Minor	Subject-III (M-3)	Open Elective (OE)	VSC, SEC (VSEC)	AEC, VEC, IKS	CC, FP, CEP, OJT/Int/RP	Cumulative Credits/Sem	Degree/ Cumulative Credit
		Mandatory (DSC)	**Elective (DSE)**	**(MIN)**							
3 (5.5)	V	DSC-11(2T) DSC-12(2T) DSC-13(2T) DSC-14(2P) DSC-15(2P)	DSE-1A/B (2T) DSE-2A/B (2P)	---	---	---	VSC-1(2T) VSC-2(2P)	---	OJT/Int (4)	22	UG Degree
	VI	DSC-16(2T) DSC-17(2T) DSC-18(2T) DSC-19(2T) DSC-20(2T) IKS DSC-21(2P) DSC-22(2P)	DSE-3A/B (2T) DSE-4A/B (2P)	---	---	---	VSC-3(2T) VSC-4(2P)	---	---	22	
	Cum . Cr.	**24**	**8**	**---**	**---**	**---**	**8**	**---**	**4**	**44**	
	Exit option: Award of UG Degree in Major with 132 credits OR Continue with Major and Minor										

Fourth Year B.Sc. (Honours)

Year (Level)	Sem	Major Core Subjects		Research Methodology (RM)	VSC, SEC (VSEC)	OE	AEC, VEC, IKS	CC, FP, CEP, OJT/Int/RP	Cumulative Credits/Sem	Degree/ Cumulative Credit
IV (6.0)	VII	DSC-23(4T) DSC-24(4T) DSC-25(4T) DSC-26(2P)	DSE-5A/B (2T) DSE-6A/B (2P)	RM(4T)	---	---	---	---	22	UG Honours Degree
	VIII	DSC-27(4T) DSC-28(4T) DSC-29(4T) DSC-30(2P)	DSE-7A/B (2T) DSE-8A/B (2P)	---	---	---	---	OJT/Int (4)	22	
	Cum. Cr.	**28**	**8**	**4**	**---**	**---**	**---**	**4**	**44**	
Four Year UG Honors Degree in Major and Minor with 176 credits										

Fourth Year B.Sc. (Honours with Research)

Year (Level)	Sem	Major Core Subjects		Research Methodology (RM)	VSC, SEC (VSEC)	OE	AEC, VEC, IKS	CC, FP, CEP, OJT/Int/RP	Cumulative Credits/Sem	Degree/ Cumulative Credit
IV (6.0)	VII	DSC-23(4T) DSC-24(4T) DSC-26(2P)	DSE-5A/B (2T) DSE-6A/B (2P)	RM(4T)	---	---	---	RP(4)	22	UG Honours with Research Degree
	VIII	DSC-27(4T) DSC-28(4T) DSC-30(2P)	DSE-7A/B (2T) DSE-8A/B (2P)	---	---	---	---	RP(8)	22	
	Cum. Cr.	**20**	**8**	**4**	**---**	**---**	**---**	**12**	**44**	
Four Year UG Honours with Research Degree in Major and Minor with 176 credits										

Sem- Semester, **DSC**- Department Specific Course, **DSE**- Department Specific Elective, **OE/GE**- Open/Generic elective, **VSC**- Vocational Skill Course, **SEC**- Skill Enhancement Course, **VSEC**- Vocation and Skill Enhancement Course, **AEC**- Ability Enhancement Course, **IKS**- Indian Knowledge System, **VEC**- Value Education Course, **T**- Theory, **P**- Practical, **CC**-Co-curricular **RM**- Research Methodology, **OJT**- On Job Training, **FP**- Field Project, **Int**- Internship, **RP**- Research Project, **CEP**- Community Extension Programme, **ENG**- English, **CI**- Constitution of India, **MIL**- Modern Indian Laguage

- Number in bracket indicate credit
- The courses which do not have practical 'P' will be treated as theory 'T'
- If student select subject other than faculty in the subjects M-1, M-2 and M-3, then that subject will be treated as Minor subject, and cannot be selected as Major at second year.

Details of T.Y. B.Sc. (Mathematics)

Course	Course Type	Course Code	Course Title	Credits	Teaching Hours/ Week			Marks			
					T	P	Total	Internal		External	
								T	P	T	P
Semester V, Level – 5.5											
DSC-11	DSC	MTH-DSC-351	Methods of Real Analysis	2	2	---	2	20	---	30	---
DSC-12	DSC	MTH-DSC-352	Modern Algebra	2	2	---	2	20	---	30	---
DSC-13	DSC	MTH-DSC-353	Integral Calculus	2	2	---	2	20	---	30	---
DSC-14	DSC	MTH-DSC-354	Practical on MTH-DSC-351	2	---	4	4	---	20	---	30
DSC-15	DSC	MTH-DSC-355	Practical on MTH-DSC-352	2	---	4	4	---	20	---	30
DSE-1A	DSE	MTH-DSE-351A	Number Theory	2	2	---	2	20	---	30	---
DSE-1B	DSE	MTH-DSE-351B	Lattice Theory	2	2	---	2	20	---	30	---
DSE-2A	DSE	MTH-DSE-352A	Practical on MTH-DSE-351A	2	---	4	4	---	20	---	30
DSE-2B	DSE	MTH-DSE-352B	Practical on MTH-DSE-351B	2	---	4	4	---	20	---	30
VSC-1	VSC	MTH-VSC-351	Programming in Python	2	2	---	2	20	---	30	---
VSC-2	VSC	MTH-VSC-352	Practical on MTH-VSC-351	2	---	4	4	---	20	---	30
OJT/Int	OJT	MTH-OJT-351	On Job Training/Internship	4	---	8	8	---	40	---	60
Semester VI, Level – 5.5											
DSC-16	DSC	MTH-DSC-361	Dynamical Systems	2	2	---	2	20	---	30	---
DSC-17	DSC	MTH-DSC-362	Linear Algebra	2	2	---	2	20	---	30	---
DSC-18	DSC	MTH-DSC-363	Metric Spaces	2	2	---	2	20	---	30	---
DSC-19	DSC	MTH-DSC-364	Laplace Transforms	2	2	---	2	20	---	30	---
DSC-20	DSC/IKS	MTH-DSC-365	Ancient Indian Mathematics	2	2	---	2	20	---	30	---
DSC-21	DSC	MTH-DSC-366	Practical on MTH-DSC-361	2	---	4	4	---	20	---	30
DSC-22	DSC	MTH-DSC-367	Practical on MTH-DSC-362	2	---	4	4	---	20	---	30
DSE-3A	DSE	MTH-DSE-361A	Partial Differential Equations	2	2	---	2	20	---	30	---
DSE-3B	DSE	MTH-DSE-361B	Computational Mathematics with SageMath	2	2	---	2	20	---	30	---
DSE-4A	DSE	MTH-DSE-362A	Practical on MTH-DSE-361A	2	---	4	4	---	20	---	30
DSE-4B	DSE	MTH-DSE-362B	Practical on MTH-DSE-361B	2	---	4	4	---	20	---	30
VSC-3	VSC	MTH-VSC-361	Optimization Techniques	2	2	---	2	20	---	30	---
VSC-4	VSC	MTH-VSC-362	Practical on MTH-VSC-361	2	---	4	4	---	20	---	30

Examination Pattern

Theory Question Paper Pattern:

- 30 (External) +20 (Internal) for 2 credits
  - External examination will be of 1½ hours duration
  - There shall be 3 questions: Q1 carrying 6 marks and Q2, Q3 carrying 12 marks each. The tentative pattern of question papers shall be as follows;
  - Q1 : Attempt any 2 out of 3 sub-questions; each 3 marks.
  - Q 2 and Q3: Attempt any 3 out of 4 sub-question; each 4 marks.

Rules of Continuous Internal Evaluation:

The Continuous Internal Evaluation for theory papers shall consist of two methods:

1. Continuous & Comprehensive Evaluation (CCE): CCE will carry a maximum of 30% weightage (30/15 marks) of the total marks for a course. Before the start of the academic session in each semester, the subject teacher should choose any three assessment methods from the following list, with each method carrying 10/5 marks:

i. Individual Assignments
ii. Seminars/Classroom Presentations/Quizzes
iii. Group Discussions/Class Discussion/Group Assignments
iv. Case studies/Case lets
v. Participatory & Industry-Integrated Learning/Field visits
vi. Practical activities/Problem Solving Exercises
vii. Participation in Seminars/Academic Events/Symposia, etc.
viii. Mini Projects/Capstone Projects
ix. Book review/Article review/Article preparation
x. Any other academic activity
xi. Each chosen CCE method shall be based on a particular unit of the syllabus, ensuring that three units of the syllabus are mapped to the CCEs.

2. Internal Assessment Tests (IAT): IAT will carry a maximum of 10% weightage (10/5 marks) of the total marks for a course. IAT shall be conducted at the end of the semester and will assess the remaining unit of the syllabus that was not covered by the CCEs. The subject teacher is at liberty to decide which units are to be assessed using CCEs and which unit is to be assessed on the basis of IAT. The overall weightage of Continuous Internal Evaluation (CCE + IAT) shall be 40% of the total marks for the course. The remaining 60% of the marks shall be allocated to the semester-end examinations. The subject teachers are required to communicate the chosen CCE methods and the corresponding syllabus units to the students at the beginning of the semester to ensure clarity and proper preparation.

Practical Examination Credit 2: Pattern (30+20)

External Practical Examination (30 marks):

- Practical examination shall be conducted by the respective department at the end of the semester.
- Practical examination will be of 3 hours duration and shall be conducted as per schedule.
- Practical examination shall be conducted for 2 consecutive days for 2 hr/ day where incubation conditionis required.
- There shall be 05 marks for journal and viva-voce. Certified journal is compulsory to appear for practical examination.

Internal Practical Examination (20 marks):

- Internal practical examination of 10 marks will be conducted by department as per schedule given.
- For internal practical examination student must produce the laboratory journal of practicals completed along with the completion certificate signed by the concerned teacher and the Head of the department.
- There shall be continuous assessment of 30 marks based on student performance throughout the semester. This assessment can include quizzes, group discussions, presentations and other activities assigned by the faculty during regular practicals. For details refer internal theory examination guidelines.
- Finally 40 (10+30) marks performance of student will be converted into 20 marks.

SEMESTER-V

T.Y. B.Sc. Mathematics (Major)
Semester-V
MTH-DSC-351: Methods of Real Analysis

Total Hours: 30 **Credits: 2**

Course Objectives	▪ To study sequences of real numbers and sequences of functions. ▪ To study the series of real numbers. ▪ To study the series of functions. ▪ To study the concept of Fourier series.	
Course Outcomes	After successful completion of this course, students are expected to: ▪ Decide, whether a given series is convergent or divergent. ▪ Use different tests for absolute convergence. ▪ Understand the concept of pointwise convergent, uniform convergent, integration and differentiation of series of functions. ▪ Understand Fourier series and half range Fourier series.	
Unit	**Contents**	**Hours**
Unit I	**Sequences of real numbers and sequences of functions** ▪ Definition of a sequence and a subsequence of real numbers. ▪ Convergence and divergence of sequences of real numbers ▪ Bounded sequences and Monotone sequences of real numbers ▪ Point wise convergence and Uniform convergence of sequences of functions ▪ Cauchy's criteria for uniform convergence of a sequence of functions ▪ Consequences of uniform convergence	**7**
Unit II	**Series of real numbers** ▪ Convergence and divergence ▪ Series with non-negative terms ▪ Alternating Series ▪ Conditional convergence and absolute convergence ▪ Rearrangement of series ▪ Tests for absolute convergence ▪ Series whose terms form non-increasing sequence	**8**
Unit III	**Series of functions** ▪ Pointwise convergence of a series of functions ▪ Uniform convergence of a series of functions ▪ Integration and differentiation of a series of functions	**7**
Unit IV	**Fourier series in the range** $[-\pi, \pi]$ ▪ Fourier series and Fourier coefficients ▪ Dirichlet's condition of convergence (statement only) ▪ Fourier series for even and odd functions ▪ Sine and Cosine Series in half range	**8**
Study Resources	▪ R.R. Goldberg, *Methods of Real Analysis*, Oxford and IBH Publishing Co.Pvt. Ltd., 2nd Edition, 1976. (Unit 1:- 2.1, 2.3, 2.4, 2.5, 2.6, 9.1, 9.2, 9.3, Unit 2:- 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, Unit 3:- 9.4, 9.5, 9.6) ▪ M. R. Spigel, *Laplace Transform and Fourier series*, Schaum series, Mc. Graw Hill, 1965. (Unit 4) ▪ S. C. Malik and Savita Arora, *Mathematical Analysis*, iv the edition New Age International Pvt. Ltd., New Delhi, 2010.	

T.Y. B.Sc. Mathematics (Major)
Semester-V
MTH-DSC-352: Modern Algebra

Total Hours: 30 **Credits: 2**

Course Objectives	▪ To gain the knowledge of basic concepts related to a group such as normal subgroups and quotient groups. ▪ To study the concepts of commutator subgroups and isomorphism of groups ▪ To understand basic concepts related to a ring such as an ideal, isomorphism of rings. ▪ To study the concepts of polynomial rings.	
Course Outcomes	After successful completion of this course, students are expected to: ▪ Know normal subgroups, simple groups and quotient groups. ▪ Know commutator subgroups, homomorphism, isomorphism of groups and Cayley's theorem. ▪ Know ideals in rings, quotient rings and isomorphism of rings. ▪ Know polynomial rings and irreducibility of polynomials.	
Unit	**Contents**	**Hours**
Unit I	**Normal subgroups and Quotient groups** ▪ Normal Subgroup ▪ Criterions for a subgroup to be a normal subgroup ▪ Union and Intersection of normal subgroup ▪ Quotient Group, Simple Group	**7**
Unit II	**Commutator subgroups and isomorphism of groups** ▪ Commutator subgroup ▪ Revision of Homomorphism and Isomorphism of Groups ▪ Isomorphism theorems for groups and examples ▪ Cayley's theorem, Theorem: $o(A_n) = \frac{o(S_n)}{2}$	**8**
Unit III	**Ideals, quotient rings and isomorphism of rings** ▪ Ring, integral domain, field and basic properties ▪ Characteristics of a ring ▪ Subrings, ideals, left ideals, right ideals, principal ideals, prime and maximal ideals. ▪ Quotient rings, Homomorphism and isomorphism of rings	**7**
Unit IV	**Polynomial rings** ▪ Polynomial rings and its properties ▪ Roots of Polynomials ▪ Factorization of Polynomials ▪ Division Algorithm for Polynomials ▪ Eisenstein's Criterion	**8**
Study Resources	▪ N. S. Gopalakrishnan, *University Algebra* (2nd Revised Edition), New Age International Publishers, 2003. **Chapter-1:** Art.-1.7, 1.8, 1.9; **Chapter-2**: Art.- 2.2, 2.3,2.4,2.5, 2.6, 2.7, 2.8, 2.9,2.14,2.15 ▪ J. B. Fraleigh, *A First Course in Abstract Algebra* (3rd Edition), Narosa Publishing House, (Tenth Reprint 2003). **Chapter-30:** Art.-30.1, 30.2, 30.3; **Chapter-31:** Art.-31.1, 31.2. ▪ N. Herstein, *Topics in Algebra* (2nd Edition), Vikas Publishing House Pvt. Ltd. New Delhi, 1964. ▪ V. K. Khanna and S. K. Bhambri, *A course in Abstract Algebra*(3rd Edition), Vikas Publishing House Pvt. Ltd. New Delhi, 2008.	

T.Y. B.Sc. Mathematics (Major)
Semester-V
MTH-DSC-353: Integral Calculus

Total Hours: 30 **Credits: 2**

Course Objectives	▪ To study Riemann Integration and its properties. ▪ To study Mean value theorems of integral calculus. ▪ To study Improper integrals with finite limit and infinite limit. ▪ To study Beta and Gamma Integrals.	
Course Outcomes	After successful completion of this course, students are expected to: ▪ Understand the concept of Riemann Integration. ▪ Know the mean value theorems of integral calculus. ▪ Understand Improper integrals with finite limit and infinite limit their properties. ▪ Learn the concepts of Beta and Gamma Integrals.	
Unit	**Contents**	**Hours**
Unit I	**Riemann integration** ▪ Definition and Existence of the Integral, The meaning of $\int_a^b f\,dx$ when $a \le b$, Inequalities for integrals ▪ Refinement of partitions ▪ Darboux's Theorem (without proof) ▪ Conditions of integrability ▪ Integrability of the sum and difference of integrable functions ▪ The integral as a limit of sum (Riemann Sums) and the limit of sum as the integral and its applications	**8**
Unit II	**Mean value theorems of integral calculus** ▪ The First mean value theorem ▪ The generalized first mean value theorem ▪ Abel's lemma (without proof) ▪ Second mean value theorem. Bonnets form and Karl Weierstrass form	**7**
Unit III	**Improper integrals** ▪ Integration of unbounded functions with finite limits of Integral ▪ Comparison Test for convergence at a of $\int_a^b f\,dx$ ▪ Convergence of the improper integrals $\int_a^b \frac{dx}{(x-a)^n}$ ▪ Cauchy's general test for convergence at the point a of $\int_a^b f\,dx$ ▪ Absolute convergence of the improper integrals $\int_a^b f\,dx$ ▪ Convergence of the integral with infinite range of Integration ▪ Comparison Test for convergence at ∞	**8**
Unit IV	**Beta and Gamma integrals** ▪ Convergence of Beta and Gamma Integrals ▪ Properties of Beta and Gamma Functions ▪ Relation between Beta and Gamma Functions ▪ Duplication Formula ▪ Evaluation of integrals using Beta and Gamma Integrals	**7**
Study Resources	▪ S. C. Malik and Savita Arora, *Mathematical Analysis*, second Edition, New Age International Pvt. Ltd., New Delhi, 2000. Chapter 9: 1 to 13, Chapter 11: 1 to 5. ▪ R.R. Goldberg, *Methods of Real Analysis*, Oxford & IBH Publishing Co. PVT. LTD, 2nd Edition, 1976.	

T.Y. B.Sc. Mathematics (Major)
Semester-V
MTH-DSC-354: Practical on MTH-DSC-351

Total Hours: 60 **Credits: 2**

Course Objectives	▪ To study sequences of real numbers and sequences of functions. ▪ To study the series of real numbers. ▪ To study the series of functions. ▪ To study the concept of Fourier series.	
Course Outcomes	After successful completion of this course, students are expected to: ▪ Decide, whether a given series is convergent or divergent. ▪ Use different tests for absolute convergence. ▪ Understand the concept of pointwise convergent, uniform convergent, integration and differentiation of series of functions. ▪ Understand Fourier series and half range Fourier series.	
Sr. No.	**Contents**	**Hours**
1	Convergence of sequence-I	**4**
2	Convergence of sequence-II	**4**
3	Pointwise convergence	**4**
4	Uniform convergence of sequence of function	**4**
5	Sequence of partial sum	**4**
6	Comparision test for series	**4**
7	Ratio test for series	**4**
8	Cauchy's condensation test for series	**4**
9	Root test for series	**4**
10	M_n test for convergence	**4**
11	Uniform convergence for series	**4**
12	Introduction to Fourier series	**4**
13	Fourier series-I	**4**
14	Fourier series-II	**4**
15	Fourier series-III	**4**

List of Practicals

Practical No: 01 Convergence of sequence-I

1) Discuss the convergence of the sequence $\left\{\frac{n+1}{n+2}\right\}$.
2) Discuss the convergence of the sequence $\left\{\frac{n^2+2}{n^4+3}\right\}$.
3) Discuss the convergence of the sequence $\left\{\frac{2n+1}{3n^2+2}\right\}$.
4) Discuss the convergence of the sequence $\{\sqrt{n}+1-\sqrt{n}\}$.
5) Discuss the convergence of the sequence $\sqrt{n}$.

Practical No: 02 Convergence of sequence-II

1) Show that the sequence $\left\{\frac{1}{n}\right\}$ converges to 0.
2) Show that the sequence $\{1\}$ converges to 1.
3) If the sequence of real $\{s_n\}$ converges to L then show that the sequence $\{|s_n|\}$ converges to $|L|$.

4) If the sequence of real $\{|s_n|\}$ converges to 0 then show that the sequence $\{s_n\}$ converges to 0.
5) Prove that limit of convergent sequence is unique.

Practical No: 03 Pointwise convergence

1) For $n \in I$, let $f_n(x) = nx(1-x^2)^n$ for $0 \leq x \leq 1$. Show that $\{f_n(x)\}$ converges to 0.
2) For $n \in I$, let $f_n(x) = nx(1-x^2)^n$ for $0 \leq x \leq 1$. Show that $\left\{\int_0^1 f_n(x)dx\right\}$ converges to $\frac{1}{2}$.
3) Let $f_n(x) = \frac{x^n}{1+x^n}$, $0 \leq x \leq 1$, show that $\{f_n(x)\}$ converges pointwise on [0,1].
4) Let $f_n(x) = \frac{\sin nx}{n}$, $0 \leq x \leq 1$, show that $\{f_n(x)\}$ converges pointwise on [0,1].
5) Let $f_n(x) = \frac{\cos nx}{n}$, $0 \leq x \leq 1$, show that $\{f_n(x)\}$ converges pointwise on [0,1].

Practical No: 04 Uniform convergence of sequence of function

1) Test the uniform convergence of $f_n(x) = x^n$, $0 \leq x \leq 1$.
2) Test the uniform convergence of $f_n(x) = \frac{x}{1+nx}$, $0 \leq x < \infty$.
3) Test the uniform convergence of $f_n(x) = \frac{nx}{1+n^2x^2}$, $-\infty < x < \infty$.
4) Test the uniform convergence of $f_n(x) = \frac{\sin nx}{n}$, $0 \leq x \leq 1$.
5) Test the uniform convergence of $f_n(x) = \frac{\cos nx}{n}$, $0 \leq x \leq 1$.

Practical No: 05 Sequence of partial sum

1) Discuss the convergence of the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$.
2) Discuss the convergence of the series $1 + 2 + 3 + 4 + \cdots$.
3) Discuss the convergence of the series $\Sigma\, x^n$.
4) Discuss the convergence of the series $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$.
5) Discuss the convergence of the series $2 + 4 + 6 + 8 + \cdots$.

Practical No: 06 Comparision test for series

1) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{n+1}{n+2}$.
2) Discuss the convergence of $\sum_{n=1}^{\infty} \frac{1+n}{1+n^2}$.
3) Discuss the convergence of $\sum_{n=1}^{\infty} \frac{n^2+2n+6}{n^3+n^2+4}$.
4) Discuss the Discuss the convergence of $\sum_{n=1}^{\infty} \frac{n^2+5}{n^4+4}$.
5) Discuss the convergence of $\sum_{n=1}^{\infty} \frac{2n^2+6n+3}{n^5+6}$.

Practical No: 07 Ratio test for series

1) Discuss the convergence of $\sum_{n=1}^{\infty} \frac{n^4}{n!}$.
2) Discuss the convergence of $\sum_{n=1}^{\infty} \frac{3}{1+2^n}$.
3) Discuss the convergence of $\sum_{n=1}^{\infty} \frac{n^n}{n!}$.
4) Discuss the convergence of $\sum_{n=1}^{\infty} \frac{n!}{n^n}$.
5) Discuss the convergence of $\sum_{n=1}^{\infty} \frac{x^n}{n!}$.

Practical No: 08 Cauchy's condensation test for series

1) Test the convergence of the series by Cauchy's condensation test $\sum_{n=1}^{\infty} \frac{1}{n}$.
2) Test the convergence of the series by Cauchy's condensation test $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

3) Test the convergence of the series by Cauchy's condensation test $\sum_{\mathrm{n}=1}^{\infty}\frac{1}{nlog\, n}$.
4) Test the convergence of the series by Cauchy's condensation test $\sum_{\mathrm{n}=1}^{\infty}\frac{1}{n\,(logn)^2}$.
5) Test the convergence of the series by Cauchy's condensation test $\sum_{\mathrm{n}=1}^{\infty}\frac{1}{n\,(logn)^x}$.

Practical No: 09 Root test for series

1) For what values of p the series $\frac{1}{1^p}-\frac{1}{2^p}+\frac{1}{3^p}-\frac{1}{4^p}+\cdots$ converges.
2) Test the convergence of $\sum_{\mathrm{n}=1}^{\infty}\left(1+\frac{1}{n}\right)^n$.
3) Test the uniform convergence of $\sum_{\mathrm{n}=1}^{\infty}\frac{\left(\frac{n+1}{n}\right)^{n^2}}{3^n}$.
4) Test the uniform convergence of $\sum_{\mathrm{n}=1}^{\infty}\frac{(n)^{n^2}}{(n+1)^{n^2}}$.
5) Test the convergence of $\sum_{\mathrm{n}=1}^{\infty}\frac{\left(1+\frac{1}{n}\right)^{2n}}{e^n}$.

Practical No:10 M_n test for convergence

1) Discuss the uniform convergence of the following series $\sum_{n=1}^{\infty}\frac{\sin nx}{n^2}$ $x\in\mathbb{R}$.
2) Discuss the uniform convergence of the following series $\sum_{n=1}^{\infty}\frac{\cos nx}{n^2}$ $x\in\mathbb{R}$.
3) Discuss the uniform convergence of the following series $\sum_{n=1}^{\infty}\frac{1}{n^2+x^2}$ $x\in[0,\infty)$.
4) Show that the series $\sum_{n=1}^{\infty}\frac{nx^2}{n^3+x^3}$ is uniformly convergent on $[0\ A]$ further show that $\lim_{x\to 1}\sum_{n=1}^{\infty}\frac{nx^2}{n^3+x^3}=\sum_{n=1}^{\infty}\frac{n}{n^3+1}$.
5) Discuss the uniform convergence of the following series $\sum_{n=1}^{\infty}e^{-nx}x^n$ $0\le x\le 10$.

Practical No:11 Uniform convergence for series

1) Discuss the uniform convergence of the following series $\sum_{n=1}^{\infty}\frac{\sin(x^2+nx^2)}{n(n+1)}$, $x\in\mathbb{R}$.
2) Discuss the uniform convergence of the following series $\sum_{n=1}^{\infty}\frac{x}{n(1+nx^2)}$, $x\in\mathbb{R}$.
3) Discuss the uniform convergence of the following series $\sum_{n=1}^{\infty}\frac{1}{n^3+n^4x^2}$, $x\in\mathbb{R}$.
4) If $\sum_{n=1}^{\infty}|a_n|<\infty$ then prove that the series $\sum_{n=1}^{\infty}a_nx^n$ is uniformly convergent for $0\le x<1$.
5) If $\sum_{n=1}^{\infty}|a_n|<\infty$ then prove that $\int_0^1\sum_{n=0}^{\infty}a_nx^n\ dx=\sum_{n=0}^{\infty}\frac{a_n}{n+1}$

Practical No:12 Introduction to Fourier series

1) Assuming that $f(x)$ can be expanded in $[-\pi,\pi]$ as $f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}^{\infty}b_n\sin nx$ prove that $\frac{1}{\pi}\int_{-\pi}^{\pi}f^2(x)dx=\frac{1}{2}{a_0}^2+\sum_1^{\infty}\left({a_n}^2+{b_n}^2\right)$.
2) Define: i) periodic function, ii) even function, iii) odd function.
3) State Fourier series.
4) State Fourier series for even function.
5) State Fourier series for odd function.

Practical No:13 Fourier series-I

1) Find the Fourier series for $(x)=\begin{cases}0, & -\pi\le\ x\le 0\\ x, & 0\le x\le\pi\end{cases}$.
2) Find the Fourier series for $(x)=\begin{cases}-1, & -\pi\le\ x\le 0\\ 1, & 0\le x\le\pi\end{cases}$.

3) Find the Fourier series for $(x) = \begin{cases} x+\pi, & 0 \le x \le \pi \\ -x+\pi, & -\pi \le x < 0 \end{cases}$.
4) Obtained the series of sine of multiple of x in $[0\ \pi]$ for $f(x) = x$.
5) Find half range cosine series for $f(x) = x$, $x \in [0\ \pi]$.

Practical No:14 Fourier series-II

1) Find the Fourier series for $f(x) = x + x^2$ on $[-\pi, \pi]$.
2) Find the Fourier series for $f(x) = |x|$ on $[-\pi, \pi]$.
3) Obtained the Fourier series for the function $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$
4) Find the half range sine series for $f(x) = \begin{cases} \frac{1}{4} - x, 0 < x < \frac{1}{2} \\ x - \frac{3}{4}, \frac{1}{2} < x < 1 \end{cases}$
5) Obtained the Fourier series for the function $f(x) = \begin{cases} 1, & -\pi \le x < 0 \\ 0, & 0 \le x < \pi \end{cases}$

Practical No:15 Fourier series-III

1) Obtained the Fourier series for the function $(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$.
2) Obtained the Fourier series for the function $(x) = \begin{cases} -x+1, & -\pi < x < 0 \\ x+1, & 0 < x < \pi \end{cases}$.
3) Obtained the Fourier series for the function $(x) = \begin{cases} 0, & -\pi \le x \le 0 \\ \sin x, & 0 \le x \le \pi \end{cases}$.
4) Obtained the Fourier series for the function $(x) = \begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2-x), & 1 \le x \le 2 \end{cases}$.
5) Find the Fourier series for $f(x) = x - x^2$ on $[-\pi, \pi]$.

T.Y. B.Sc. Mathematics (Major)
Semester-V
MTH-DSC-355: Practical on MTH-DSC-352

Total Hours: 60 **Credits: 2**

Course Objectives	▪ To gain the knowledge of basic concepts related to a group such as normal subgroups and quotient groups. ▪ To study the concepts of commutator subgroups and isomorphism of groups ▪ To understand basic concepts related to a ring such as an ideal, isomorphism of rings. ▪ To study the concepts of polynomial rings.	
Course Outcomes	After successful completion of this course, students are expected to: ▪ Understand and solve problems on normal subgroups and simple group. ▪ Explain the concepts and solve the problems on subrings and ideals in rings ▪ Apply isomorphism theorems to solve problems on isomorphic groups and rings. ▪ Understand the concepts of reducible and irreducible polynomials in polynomial rings.	
Sr. No.	**Contents**	**Hours**
1	Normal Subgroups-I	**4**
2	Normal Subgroups-II	**4**
3	Quotient Groups	**4**
4	Groups	**4**
5	Isomorphism Theorems for Groups-I	**4**
6	Isomorphism Theorems for Groups-II	**4**
7	Isomorphism Theorems for Groups-III	**4**
8	Isomorphism Theorems for Groups-IV	**4**
9	Subrings	**4**
10	Ideals of Ring-I	**4**
11	Ideals of Ring-II	**4**
12	PID and Characteristic of Ring	**4**
13	Fields and Quotient Rings	**4**
14	Polynomial Rings	**4**
15	Reducible and Irreducible Polynomials	**4**

List of Practicals

Practical No. - 1: Normal Subgroups-I

1. Let $G = (\{1, -1, i, -i\}, \times)$, be a group and $H = (\{1, -1\}, \times)$ be a subgroup of G. Show that H is normal in G.
2. Prove that every subgroup of an abelian group is normal.
3. Prove that every subgroup of a cyclic group is normal.
4. Prove that intersection of two normal subgroups of a group G is normal in G.
5. Show by an example that union of two normal subgroups of a group G need not be a normal subgroup.

Practical No. - 2: Normal Subgroups-II

1. If H is a subgroup of a group G and $N(H) = \{g \in G : gHg^{-1} = H\}$ then show that $N(H)$ is a subgroup of G and H is normal in $N(H)$.
2. Show that $Z(G)$ is a normal subgroup of a group G.

3. Find all normal subgroups of the group of quaternion's $Q = \{\pm 1, \pm i, \pm j, \pm k\}$.
4. Give an example of subgroups H, K of G such that H is normal in K and K normal in G but H is not normal in G.
5. If $G = (\mathbb{Z}, +)$ and $N = (3\mathbb{Z}, +)$, then find the quotient group $\frac{G}{H}$.

Practical No. - 3: Quotient Groups

1. Let $G = (\mathbb{Z}_{12}, +_{12})$ be a group and $H = (\{\bar{0}, \bar{3}, \bar{6}, \bar{9}\}, +_{12})$ be a subgroup of G. Show that $\frac{G}{H}$ is a group.
2. Prove that the quotient group of an abelian group is abelian.
3. Give an example of group G, such that quotient group $\frac{G}{H}$ is abelian group but G is not abelian group.
4. Prove that the quotient group of a cyclic group is cyclic.
5. Give an example of group G, such that quotient group $\frac{G}{H}$ is cyclic group but G is not cyclic group.

Practical No. - 4: Groups

1. Let $G = \mathrm{GL}(2, \mathbb{R}) = \{A : A$ is non-singular 2×2 matrix over $\mathbb{R}\}$, a group under usual matrix multiplication and $H = \mathrm{SL}(2, \mathbb{R}) = \{A \in G : |A| = 1\}$ a subgroup of G. Show that H is normal in G.
2. Let $G = (\{1, -1, i, -i, j, -j, k, -k\}, \cdot)$ be a group of quaternions and $S = \{i, j\}$. Show that S generates G.
3. Let $S = \{(1,2), (1,3)\} \subseteq S_3$. Find $\langle S \rangle$.
4. If $= S_3$, then find G'.
5. Show that $(\mathbb{Z}_7, +_7)$ is a simple group.

Practical No. - 5: Isomorphism Theorems for Groups-I

1. If $f: (G, *) \to (G_1, \odot)$ is a group homomorphism, then show that $\ker(f)$ is a normal subgroup of G.
2. Let $f: (G, *) \to (G_1, \odot)$ be an one-one group homomorphism. Prove that $\ker(f) = \{0\}$.
3. Let $f: (G, *) \to (G_1, \odot)$ be a group homomorphism with $\ker(f) = \{0\}$. Prove that f is one-one.
4. If $f: G \to G_1$ is an on to group homomorphism, then prove that $\frac{G}{\ker(f)} \cong G_1$.
5. Let H and K be subgroups of a group G and H be a normal subgroup of G. Prove that $\frac{HK}{H} \cong \frac{K}{H \cap K}$

Practical No. - 6: Isomorphism Theorems for Groups-II

1. Let H and K be normal subgroups of a group G such that $H \subseteq K$. Prove that $\frac{G}{K} \cong \frac{G/H}{K/H}$.
2. Show that A_n is a normal subgroup of S_n , where $n \geq 2$.
3. If H is a subgroup of a group S_n $(n \geq 2)$, then show that either all permutations in H are even or exactly half of them are even.
4. Show that every infinite cyclic group is isomorphic to $(\mathbb{Z}, +)$.
5. Show that every finite cyclic group of order n is isomorphic to $(\mathbb{Z}_n, +_n)$.

Practical No. - 7: Isomorphism Theorems for Groups-III

1. Find how many onto group homomorphisms are there from $\mathbb{Z}_{12}$ to $\mathbb{Z}_5$?
2. Let $\mathbb{R}^*$ be the multiplicative group of non-zero reals. Show that $\frac{\mathrm{GL}(2,\mathbb{R})}{\mathrm{SL}(2,\mathbb{R})} \cong \mathbb{R}^*$.
3. Let $G = \{1, -1\}$ be the group under multiplication. Show that the function $f: S_n \to G$ defined by $f(\sigma) = \begin{cases} 1 & \text{if } \sigma \text{ is even} \\ -1 & \text{if } \sigma \text{ is odd} \end{cases}$, is an onto group homomorphism. Find its kernel.
4. Show that $\mathbb{Z}_9$ is not a homomorphic image of $\mathbb{Z}_{16}$.
5. Show that the group $(\mathbb{Q}, +)$ is not isomorphic to $(\mathbb{Q}^+, \cdot)$.

Practical No. - 8: Isomorphism Theorems for Groups-IV

1. Prove that there are only two (upto isomorphisms) groups of order six.
2. Show that every group is isomorphic to permutation group.
3. Let G be an abelian group. Show that a map $f: G \to G$ defined by $f(x) = x^{-1}$, for all $x \in G$, is an automorphism.
4. Let G be an group and a map $f: G \to G$ defined by $f(x) = x^{-1}$, for all $x \in G$, be an automorphism. Show that G is abelian.
5. Let G be a finite abelian group of order $2k + 1$ $(k > 0)$. Show that G is non-trivial homomorphism.

Practical No. - 9: Subrings

1. Let R be a ring and $Z(R) = \{x \in R: xy = yx,\ \forall y \in R\}$. Show that $Z(R)$ is a subring of R.
2. Prove that intersection of two subrings of a ring R is again a subring of R.
3. Show by an example that union of two subrings of a ring is not a subring.
4. Let R be a ring and $a \in R$. Show that $S = \{x \in R : xa = 0\}$ is a subring of R.
5. Prove or disprove: every subring of ring R is an ideal of R.

Practical No. - 10: Ideals of Ring-I

1. If I, J are ideals of a ring R, then prove that $I + J$ is an ideal of R.
2. Show that $n\mathbb{Z}$ is an ideal of the ring $(\mathbb{Z}, +, \cdot)$ where $n \in \mathbb{N}$.
3. Show by an example that union of two ideals of a ring R is not an ideal of R.
4. Show that intersection of two prime ideals in a ring need not be a prime ideal.
5. Find all prime ideals and maximal ideals in the ring $(\mathbb{Z}_{12}, +_{12}, \times_{12})$.

Practical No. - 11: Ideals of Ring-II

1. Prove that p is a prime integer if and only if $p\mathbb{Z}$ is a prime ideal in ring $\mathbb{Z}$.
2. Prove that an ideal I of $\mathbb{Z}$ is a prime ideal if and only if $I = p\mathbb{Z}$ for some prime p or $I = \{0\}$.
3. Let $R = C[0, 1] = \{f : f$ is a continuous real valued function defined on $[0, 1]\}$ be a ring under the operations $(f + g)(x) = f(x) + g(x)$ and $(f \cdot g)(x) = f(x)g(x)$ and $(\mathbb{R}, +, \cdot)$ be the ring of real under usual addition and multiplication. Show that $\left\{f \in R : f\left(\frac{1}{2}\right) = 0\right\}$ is a maximal ideal of R.
4. Show that $\{0\}$, R are the only ideals in a field R.
5. Let R be a finite commutative ring with identity 1 and I be an ideal of R. Prove that I is prime ideal of R if and onlyu if I is a maximal ideal of R.

Practical No. - 12: PID and Characteristics of Ring

1. Prove that $\mathbb{Z}[i]$ is an integral domain.
2. Show that $(\mathbb{Z}, +, \cdot)$ is a PID.
3. Show that characteristics of a Boolean ring is two.
4. Find the characteristics for the rings i) $(\mathbb{Z}_n, +_n, \times_n)$ ii) $(\mathbb{Z}, +, \cdot)$.
5. Prove that characteristic of an integral domain is either 0 or a prime number.

Practical No. - 13: Fields and Quotient Rings

1. If $f : R \to R'$ is an onto ring homomorphism, then R' is isomorphic to some quotient ring of R.
2. Let R be a division ring and $Z(R) = \{x \in R: xy = yx\ \ \forall y \in R\}$. Show that $Z(R)$ is a field.
3. Show that $\mathbb{Z}_n \cong \frac{\mathbb{Z}}{n\mathbb{Z}}$ where $\mathbb{Z}$ is the ring of integers.
4. Find the field of quotients of $\mathbb{Z}[i]$.
5. Prove that quotient ring of a commutative ring is commutative.

Practical No. - 14: Polynomial Rings

1. Let $f(x) = 2x^3 + 4x^2 + 3x + 2$ and $g(x) = 3x^4 + 2x + 4$ in $\mathbb{Z}_5[x]$. Find

a. a) $f(x) + g(x)$ b) $f(x) \cdot g(x)$ c) $\deg(f(x) \cdot g(x))$.

2. Let $f(x) = x^6 + 3x^5 + 4x^2 - 3x + 2$ and $g(x) = x^2 + 2x - 3$ be polynomials in $\mathbb{Z}_7[x]$. Find $q(x)$, $r(x) \in \mathbb{Z}_7[x]$ such that $f(x) = g(x).q(x) + r(x)$ with $\deg(r(x)) < 2$.
3. a) Find all zeros of $f(x) = x^5 + 3x^3 + x^2 + 2x$ in $\mathbb{Z}_5$.
   b) Examine whether the polynomial $x^3 + 3x^2 + x - 4$ is irreducible over the field $(\mathbb{Z}_7, +_7, \times_7)$.
4. Express the polynomial $x^4 + 4$ as a product of linear factors in $\mathbb{Z}_5[x]$.
5. Give an example of polynomials $f(x)$ and $g(x)$ in a ring $\mathbb{Z}_6[x]$ such that$\deg(f(x) \cdot g(x)) <$ $\deg(f(x)) + \deg(g(x))$.

Practical No. - 15: Reducible and Irreducible Polynomials

1. Using Eisenstein's criteria show that the following polynomials are irreducible over field of rationals. a) $x^2 + 8x - 2$b) $8x^3 + 6x^2 - 9x + 24$.
2. Prove that the polynomial $1 + x + x^2 + \cdots + x^{p-1}$ is irreducible over field of rationals, where p is a prime number.
3. Show that $\frac{\mathbb{Z}_5[x]}{\langle x^3+3x+2\rangle}$ is a field.
4. Show that $\frac{\mathbb{Z}_3[x]}{\langle x^3+x+1\rangle}$ is not an integral domain.
5. Show that $\langle x^2 + 1\rangle$ is not a prime ideal of $\mathbb{Z}_2[x]$.

T.Y. B.Sc. Mathematics (Elective)
Semester-V
MTH-DSE-351A: Number Theory

Total Hours: 30 **Credits: 2**

Course Objectives	▪ To know prime numbers and Diophantine equations. ▪ To study congruences and its properties. ▪ To study perfect numbers, mersenne numbers and Fermat's numbers. ▪ To know the concept of Fibonacci numbers and finite continued fractions	
Course Outcomes	After successful completion of this course, students are expected to: ▪ Understand prime numbers and their properties and also able to find solution of Diophantine equations. ▪ Able to solve the real life problems by using congruences. ▪ Able to distinguish perfect numbers, mersenne numbers and Fermat's numbers. ▪ Use Fibonacci numbers to real life problems and learn basic properties of Finite continued fractions.	
Unit	**Contents**	**Hours**
Unit I	**Prime numbers and Diophantine Equation:** ▪ The Fundamental Theorem of Arithmetic ▪ The Sieve of Eratosthenes ▪ The Goldbach Conjecture ▪ The Diophantine Equation $ax + by = c$.	**7**
Unit II	**The theory of Congruences:** ▪ Basic Properties of Congruence ▪ Binary and decimal representations of integers ▪ Special Divisibility Tests ▪ Linear Congruences and the Chinese Remainder Theorem ▪ Fermat's Factorization Method ▪ The Fermat's Little Theorem and pseudoprimes ▪ Wilson's Theorem	**8**
Unit III	**Perfect numbers:** ▪ Perfect Numbers ▪ Mersenne Numbers ▪ Fermat's Numbers	**7**
Unit IV	**Fibonacci numbers and Finite continued fractions:** ▪ The Fibonacci Sequence ▪ Certain Identities Involving Fibonacci Numbers ▪ Finite continued fractions	**8**
Study Resources	▪ David M. Burton, *Elementary Number Theory*, (Sixth Edition), Tata McGraw-Hill Edition, New Delhi, 2007. Ch.3 : 3.1 to 3.3, Ch . 2 : 2.5, Ch.4 : 4.2 to 4.4, Ch.5 : 5.2 to 5.4, Ch.11 : 11.2 to 11.4, Ch 14 : 14.2 to 14.3, Ch 15: 15.2 ▪ T. M. Apostol, *Introduction to Analytic Number Theory*, Springer International student Edition, 1972. ▪ Hari Kishan, *Number Theory*, Krishna Prakashan Media (p) Ltd, Meerat, 2019.	

T.Y. B.Sc. Mathematics (Elective)
Semester-V
MTH-DSE-351B: Lattice Theory

Total Hours: 30 **Credits: 2**

Course Objectives	▪ To study the concept of a poset. ▪ To study the concept of a lattice and its diagrammatic representation. ▪ To study the concept of ideals and its properties. ▪ To study modular, distributive lattice and their inter-relation.	
Course Outcomes	After successful completion of this course, students are expected to: ▪ Understand the structure of poset. ▪ Represent lattice in diagrammatic form. ▪ Learn the concepts of ideals and their properties as well as the concepts of homomorphism. ▪ Understand modular, distributive lattice and their interrelation.	
Unit	**Contents**	**Hours**
Unit I	**Posets** ▪ Posets and Chains ▪ Diagrammatical Representation of posets ▪ Maximal and Minimal elements of subset of a poset, Zorn's Lemma (Statement only) ▪ Supremum and infimum ▪ Poset isomorphism, Duality Principle.	**7**
Unit II	**Lattices** ▪ Two definitions of lattice and equivalence of two definitions ▪ Modular and Distributive inequalities in a lattice ▪ Sublattice and Semilattice ▪ Complete lattice	**8**
Unit III	**Ideals and homomorphisms** ▪ Ideals, Union and intersection of Ideals ▪ Prime Ideals, Principal Ideals ▪ Dual Ideals, Principal dual Ideals ▪ Complements, Relative Complements ▪ Homomorphisms, Join and meet homomorphism	**7**
Unit IV	**Modular and distributive lattices** ▪ Modular lattice ▪ Distributive lattice ▪ Sublattice of Modular lattice ▪ Homomorphic image of Modular lattice ▪ Complemented and Relatively complemented lattice	**8**
Study Resources	▪ Vijay K.Khanna, Lattices and Boolean Algebra, (Second Edition), Vikas Publ. Pvt. Ltd., 2004. (Chapter-2: Art.-30.1, 30.2, 30.3; Chapter-3: Art.-31.1, 31.2; Chapter-4: Art.-31.1, 31.2.) ▪ George Gratzer, General Lattice Theory, (Second Edition), Birkhauser, 2013.	

T.Y. B.Sc. Mathematics (Elective)
Semester-V
MTH-DSE-352A: Practical on MTH-DSE-351A

Total Hours: 60 **Credits: 2**

Course Objectives	▪ To know prime numbers and Diophantine equations. ▪ To study congruences and its properties. ▪ To study perfect numbers, mersenne numbers and Fermat's numbers. ▪ To know the concept of Fibonacci numbers and finite continued fractions
Course Outcomes	After successful completion of this course, students are expected to: ▪ Understand prime numbers and their properties and also able to find solution of Diophantine equations. ▪ Able to solve the real life problems by using congruences. ▪ Able to distinguish perfect numbers, mersenne numbers and Fermat's numbers. ▪ Use Fibonacci numbers to real life problems and learn basic properties of Finite continued fractions.

Sr. No.	Contents	Hours
1	Prime numbers	**4**
2	Composite numbers	**4**
3	The Sieve of Eratosthenes and Goldbach Conjecture	**4**
4	Diophantine Equations	**4**
5	Congruences	**4**
6	Binary and decimal representations of integers, Special Divisibility Tests	**4**
7	Linear Congruences	**4**
8	The Fermat's Little Theorem, pseudoprimes and Wilson's Theorem	**4**
9	Perfect numbers	**4**
10	Mersenne Numbers	**4**
11	Fermat's Numbers	**4**
12	Fibonacci numbers	**4**
13	Continued fractions	**4**
14	k^{th} convegent	**4**
15	Applications of continued fractions to Diophantine Equations	**4**

List of Practicals

Practical No. 1: Prime numbers

1. Write ppf of 2800.
2. Show that $\sqrt{7}$ is an irrational number.
3. Show that any prime number of the form $3p + 1$ is of the form $6m + 1$ where $p, m \in \mathbb{N}$.
4. Show that the only prime number of the form $n^3 - 1$ is 7.
5. Prove that there are infinitely prime numbers of the form $4k + 3$.

Practical No. 2: Composite numbers

1. If $p \geq 5$ is a prime, then show that $p^2 + 2$ is composite.
2. If $p \neq 5$ is an odd prime, prove that either $p^2 - 1$ or $p^2 + 1$ is divisible by 10.
3. Prove that the product of two integers of the form $6n + 5$ is of the form $6m + 1$.

4. Assuming that P_n is the n^{th} prime number, establish that the sum $\frac{1}{P_1}+\frac{1}{P_2}+\cdots+\frac{1}{P_n}$ is never an integer.
5. Find the smallest positive integer n such that $f(n)=n^2+21n+1$ is a composite number.

Practical No. 3: The Sieve of Eratosthenes and Goldbach Conjecture

1. Determine whether the integer 769 is prime by testing all primes $p \leq \sqrt{769}$ as possible divisors. Do the same for integer 1009
2. Obtain all prime numbers between 2 and 100 by Siev of Eratosthenes method.
3. Find all prime numbers between 101 and 150 by Siev of Eratosthenes method.
4. Obtain all prime numbers between 121 and 170 by Siev of Eratosthenes method.
5. Show that the sum of twin primes p and $p+2$ is divisible by 12 where $p>3$.

Practical No. 4: Diophantine Equations

1. Determine the solutions in the integers of $56x+72y=40$.
2. Determine integer solutions of the equation $221x+35y=11$.
3. Determine all solutions in positive integers of $54x+21y=906$.
4. Determine all solutions in the positive integers of the Diophantine equation $18x+5y=48$.
5. Let n be the number of coins. If you make 77 strings of n, then 50 coins are short, but if you make 78 strings of n, it is exact. Find the smallest value of n.

Practical No. 5: Congruences

1. Show that $41 \mid (2^{20}-1)$.
2. Find the remainder when 2^{50} is divided by 7.
3. Give numbers a, b, k, n such that $ka \equiv kb (mod\ n)$ but $a \not\equiv b (mod\ n)$.
4. Find the remainder when $1! + 2! + 3! + 4! + \cdots + 99! + 100!$ is divided by 12.
5. What is the remainder when $1^5+2^5+3^5+4^5+\cdots+99^5+100^5$ is divided by 4?

Practical No. 6: Binary and decimal representations of integers, Special Divisibility Tests

1. Write the representation of 105 with base 3.
2. Find the value of $(14313)_5$.
3. Find the last two digit number of the number 9^{9^9}.
4. Without performing the division, determine whether the integer 176521221 is divisible by 9.
5. Without performing the division, determine whether the integer 82617381 is divisible by 11.

Practical No. 7: Linear Congruences

1. Solve $34x \equiv 60\ (mod\ 98)$.
2. Solve $6x \equiv 15\ (mod\ 21)$.
3. Does the linear congruence $36x \equiv 16 (mod\ 108)$ have a solution.
4. Find an integer which leaves remainder 5 when divided by 11 and 2 when divided by 19.
5. Solve the following system of linear congruences: $x \equiv 3 (mod\ 5)$, $x \equiv 5\ (mod\ 7)$, $x \equiv 7 (mod\ 9)$.

Practical No. 8: The Fermat's Little Theorem, pseudoprimes and Wilson's Theorem

1. Factorize 3901 by Fermat's factorization method.
2. Show that 341 is a pseudo prime number.
3. If $7 \nmid a$, then show that either a^3-1 or a^3+1 is divisible by 7.
4. Find the remainder when 11^{104} is divided by 17.
5. Show that $18! \equiv -1\ (mod\ 437)$.

Practical No. 9: Perfect numbers

1. Show that the integer $n = 2^{10}(2^{11} - 1)$ is not perfect number.
2. If n is even perfect number then prove that $\sum_{d|n} \frac{1}{d} = 2$.
3. Show that a perfect square can not be a perfect number.
4. Let n be an even perfect number > 6. Show that the sum of digits of n is congruent to 1 modulo 9.
5. Show that the product of two odd prime numbers is not a perfect number.

Practical No. 10: Mersenne Numbers

1. Find the Smallest Divisor of M_{11}.
2. Show that M_{29} is a composite number.
3. From the congruence $5.2^7 \equiv -1 \ (mod\ 641)$, deduce that $2^{32} + 1 \equiv 0 \ (mod\ 641)$. Hence $641 \mid F_5$.
4. Let $p, q = 2p + 1$ be prime numbers. Show that $q \mid M_p$ or $q \mid (M_p + 2)$ but not both.
5. Prove that the Mersenne number M_{13} is prime. Hence show that $n = 2^{12}(2^{13} - 1)$ is perfect.

Practical No. 11: Fermat's Numbers

1. Find first five Fermat's numbers.
2. For $m \neq n$, prove that $(F_m, F_n) = 1$.
3. Show that F_5 is a composite number.
4. From the congruence $5.2^7 \equiv -1 \ (mod\ 641)$, deduce that $2^{32} + 1 \equiv 0 \ (mod\ 641)$.
5. For $n \geq 2$, show that the last digit of Fermat number $F_n = 2^{2^n} + 1$ is 7.

Practical No. 12: Fibonacci numbers

1. Show that $(u_n, u_{n+2}) = 1$ where $n \in \mathbb{N}$.
2. Prove that $u_m \mid u_n$ if and only if $m \mid n$.
3. Prove that gcd of two Fibonacci numbers is again a Fibonacci number.
4. Prove that $u_{n+5} \equiv 3u_n (mod\ 5)$.
5. If $3|u_n$, then show that $9|\{(u_{n+1})^3 - (u_{n-1})^3\}$.

Practical No. 13: Finite continued fractions

1. Express $\frac{187}{57}$ as finite simple continued fraction.
2. Express $\frac{19}{51}$ as finite simple continued fraction.
3. Express $\frac{-19}{51}$ as finite simple continued fraction.
4. Determine the rational number represented by $[-2 : 2, 4, 6, 8]$ as a finite simple continued fraction.
5. Determine the rational number represented by $[4 : 2, 1, 3, 1, 2, 4]$ as a finite simple continued fraction.

Practical No. 14: k^{th} convergent

1. Find all k^{th} convergent of $\frac{19}{51}$.
2. Find all k^{th} convergent of $\frac{-19}{51}$.
3. Find all k^{th} convergent of $[1: 2, 3, 3, 2, 1]$.
4. Find all k^{th} convergent of $[-3: 1, 1, 1, 1, 3]$.
5. Find all k^{th} convergent of $[-2 : 2, 4, 6, 8]$.

Practical No. 15: Applications of continued fractions to Diophantine Equations

1. Represent $[-1: 2,1,6,1]$ as an odd number of partial denominators.
2. By using simple continued fraction, solve the Diophantine equation $19x + 51y = 1$.

3. By using simple continued fraction, solve the Diophantine equation $18x + 5y = 24$.
4. By using simple continued fraction, solve the Diophantine equation $123x + 360y = 99$.
5. By using simple continued fraction, solve the Diophantine equation $56x + 72y = 40$.

T.Y. B.Sc. Mathematics (Elective)
Semester-V
MTH-DSE-352B: Practical on MTH-DSE-351B

Total Hours: 60 **Credits: 2**

Course Objectives	▪ To study the concept of a poset. ▪ To study the concept of a lattice and its diagrammatic representation. ▪ To study the concept of ideals and its properties. ▪ To study modular, distributive lattice and their inter-relation.	
Course Outcomes	After successful completion of this course, students are expected to: ▪ Understand the structure of poset. ▪ Represent lattice in diagrammatic form. ▪ Learn the concepts of ideals and their properties as well as the concepts of homomorphism. ▪ Understand modular, distributive lattice and their interrelation.	
Sr. No.	**Contents**	**Hours**
1	Poset-I	**4**
2	Poset-II	**4**
3	Diagrammatic Representation-I	**4**
4	Diagrammatic Representation-II	**4**
5	Lattices-I	**4**
6	Lattices-II	**4**
7	Sublattices	**4**
8	Semilattices	**4**
9	Ideals-I	**4**
10	Ideals-II	**4**
11	Complements	**4**
12	Homomorphisms	**4**
13	Modular lattices-I	**4**
14	Modular lattices-II	**4**
15	Distributive lattices	**4**

List of Practicals

Practical No. 1: Poset-I

1. Show whether the relation $(x, y) \in R$, if $x \geq y$ defined on the set of positive integers is a partial order relation.
2. Show that the relation 'Divides' defined on $\mathbb{N}$ is a partial order relation.
3. Show that in a poset P, $a < a$ for no $a \in P$ and $a < b, b < c \Rightarrow a < c$.
4. Prove that intersection of two partial ordered relations on a non-empty set P is again a partial ordered relation on P.
5. Show that the inclusion relation R given by $x \subseteq y$ is a partial ordering on the power set of a set A.

Practical No. 2: Poset-II

1. Show that the relation R where $(x, y) \in R$ such that $x < y$ defined on N, the set of all positive integers is neither a partially ordered relation nor an equivalence relation but is a total order relation.
2. Show that $\mathbb{N}$ under ‘usual less than or equal to relation’ forms a poset.

3. Let A, B be posets. Denote $A \times B = \{(a, b): a \in A, b \in B\}$. For $(a_1, b_1), (a_2, b_2) \in A \times B$, define $(a_1, b_1) \leq (a_2, b_2) \Leftrightarrow a_1 \leq a_2$ in A and $b_1 \leq b_2$ in B. Show that $(A \times B, \leq)$ is a poset.
4. If (P, ρ) and (P, σ) are poset, then show that $(P, \rho \cap \sigma)$ is also a poset.
5. List all chains containing two elements in a poset $\{1, 2, 3, 6\}$ under divisibility relation.

Practical No. 3: Diagrammatic Representation-I

1. Draw the Hass diagram for the poset $\{1, 2, 5, 6\}$ under usual less than or equal to relation.
2. Draw the Hass diagram for the poset $\{1, 2, 5, 6\}$ under divisibility relation.
3. Draw the Hasse diagram representing the partial ordering $\{(A, B)|\ A \subseteq B\ \}$ on the power set $\mathcal{P}(S)$ where $S = \{1, 2, 3\}$.
4. Draw the Hasse diagram of D_{15}.
5. Draw the Hasse diagram of D_{30}.

Practical No. 4: Diagrammatic Representation-II

1. Find all maximal elements and minimal elements in the poset $\{2, 3, 4, 6\}$ under divisibility relation.
2. If S is non-empty finite subset of a poset P, then prove that S has maximal and minimal elements.
3. Let P be the poset $\{2, 3, 4, 6\}$ under divisibility relation. Show that P is self dual.
4. Prove that the two chains S and T under usual less than or equal to relation are dually isomorphic where $S = \left\{0, \dots, \frac{1}{n}, \dots, \frac{1}{3}, \frac{1}{2}, 1\right\}, T = \left\{0, \frac{1}{2}, \frac{2}{3}, \ \dots, \frac{n}{n+1}, \dots, 1\right\}$.
5. Let P, Q be posets. Prove that a function $f: P \rightarrow Q$ is an isomorphism if and only if f is isotone and has an isotone inverse.

Practical No. 5: Lattices-I

1. Show that every chain is a lattice.
2. Show by an example that product of two chains need not be a chain.
3. Draw the lattice diagram of D_{20} and show that it is same as the product of two chains with three and two elements.
4. Let $(L, \leq)$ be a lattice and $a, b, c, d \in L$. If $\leq b, c \leq d$, then prove that $a \wedge c \leq b \wedge d$ and $a \vee c \leq b \vee d$.
5. Let $(L, \leq)$ be a lattice and $a, b, c \in L$. Prove that $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$.

Practical No. 6: Lattices-II

1. Let $(L, \leq)$ be a lattice and $a, b, c \in L$. Prove that $a \vee (b \wedge c) \geq (a \vee b) \wedge (a \vee c)$.
2. Let $(L, \leq)$ be a lattice and $a, b, c \in L$. Show that $(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) \leq (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$.
3. If C_1 and C_2 are two chains with two or more elements, then show that $C_1 \times C_2$ is not a chain.
4. Let $(L, \leq)$ be a lattice and $a, b, c \in L$. If $a \geq b$, then prove that $a \wedge (b \vee c) \geq b \vee (a \wedge c)$.
5. Give an example of a poset which is not a lattice.

Practical No. 7: Sublattices

1. Prove that every non-empty subset of chain is a sublattice.
2. Show that a non-empty intersection of two sublattices of a lattice L is again a sublattice of L.
3. Show by an example that intersection of two sublattices of a lattice L may not be a sublattice of L.
4. Show by an example that union of two sublattices of a lattice L may not be a sublattice of L.
5. Prove that a lattice L is a chain if and only if every non-empty subset of L is a sublattice.

Practical No. 8: Semilattices

1. Prove that dual of a complete lattice is a complete lattice.
2. Which of the following lattices is not a complete lattice?

    i) $\{1, 2, 3, \dots, 10\}$under usual less than or equal to relation.
    ii) $\mathbb{Z}$under usual less than or equal to relation.
3. Give an example of a join semilattice which is not a meet semilattice.
4. Give an example of a meet semilattice which is not a join semilattice.
5. Give an example of a meet semilattice which is a join semilattice.

Practical No. 9: Ideals-I

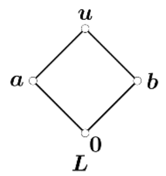
1. Show that every ideal of a lattice L is a sublattice of L.
2. Show that intersection of two ideals of a lattice L is an ideal of L.
3. Show by an example that union of two dual ideals of a lattice L need not be an ideal of L.
4. Prove that every dual ideal of a lattice L is a sublattice of L.
5. Give an example of a sublattice of a lattice D_{10} which is not an ideal of D_{10}.

Practical No. 10: Ideals-II

1. Show that intersection of two dual ideals of a lattice L is a dual ideal of L.
2. In a finite lattice L, prove that every ideal is a principal ideal.
3. Find all prime ideals in a lattice D_{18}.
4. Show by an example that union of two dual ideals of a lattice L need not be a dual ideal of L.
5. Let I be a prime ideal of a lattice L. Prove that $L - I$ is a dual prime ideal of L.

Practical No. 11: Complements

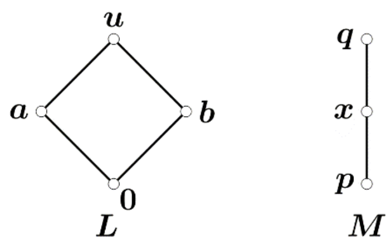
1. Show that the following lattice is a complemented lattice.



2. Show that the lattice D_{10} is a complemented lattice.
3. Prove that two bounded lattices A, B are complemented if and only if $A \times B$ is complemented.
4. Prove that dual of complemented lattioce is complemented.
5. Prove that homomorphic image of a relatively complemented lattice is relatively complemented.

Practical No. 12: Homomorphisms

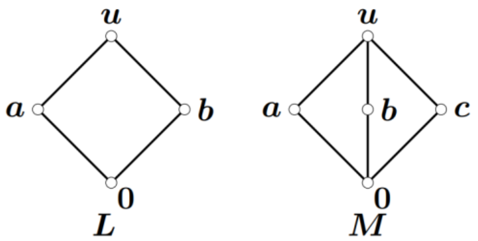
1. Show that every join homomorphism preserves order.
2. Show that every meet homomorphism preserves order.
3. Let L, M be lattices and $0'$ be the least element in M. If $\theta: L \to M$ is an onto homomorphism, then prove that $Ker\theta$ is an ideal of L.
4. Let L, M be lattices and $\theta: L \to M$ be an onto homomorphism. If 0 is the least element in L, then, prove that $\theta(0)$ is the least element in M.
5. Let L, M be lattices as given below.



Let $\phi: L \to M$ be defined by $\phi(0) = p,\ \phi(a) = x, \phi(b) = x, \phi(u) = q$. Show that ϕ is neither a meet nor a join homomorphism.

Practical No. 13: Modular lattices-I

1. Show that every chain is a modular lattice.
2. Prove that Pentagonal lattice is not modular.
3. Show that the following lattices are modular.



4. Prove that sublattice of a modular lattice is modular.
5. If a, b, c are elements of a modular lattice L with greatest element u such that $a \vee b = (a \wedge b) \vee c = u$, then show that $a \vee (b \wedge c) = b \vee (c \wedge a) = c \vee (a \wedge b) = u$.

Practical No. 14: Modular lattices-II

1. Prove that homomorphic image of a modular lattice is modular.
2. Prove that every complemented modular lattice is relatively complemented.
3. Show that two lattice L and M are modular if and only if $L \times M$ is modular lattice.
4. Prove that every distributive lattice is modular.
5. Show by an example that every modular lattice need not be distributive.

Practical No. 15: Distributive lattices

1. Show that every chain is a distributive lattice.
2. Prove that sublattice of a distributive lattice is distributive.
3. Prove that homomorphic image of a distributive lattice is distributive.
4. Prove that a lattice L is distributive if and only if whenever $a, b, c \in L$ such that $a \wedge c = b \wedge c$ and $a \vee c = b \vee c \Rightarrow \; a = b$.
5. If an element x has a complement in a distributive lattice L and $x \in [a, b] \subseteq L$, then show that x has a relative complement in $[a, b]$.

T.Y. B.Sc. Mathematics (Vocational)
Semester-V
MTH-VSC-351: Programming in Python

Total Hours: 30 **Credits: 2**

Course Objectives	▪ To understand conditional statements. ▪ To study iterators and modules. ▪ To know the plotting of various graphs and charts using Python Matplotlib. ▪ To know the applications of NumPy and SciPy for solving problems in the subjects of curriculum.	
Course Outcomes	After successful completion of this course, students are expected to: ▪ create and run Python programs using conditional statements. ▪ use iterators and modules for solving mathematical problems. ▪ apply Python Matplotlib to plot various graphs and charts. ▪ apply NumPy and SciPy for solving problems in the subjects of curriculum.	
Unit	**Contents**	**Hours**
Unit I	**Conditional Statements** ▪ Conditional and alternative statements ▪ Chained and nested conditionals (if, if-else, if-elif-else, nested if, nested if-else) ▪ Looping Statements (while, for) ▪ Python Functions ▪ Python Classes and Objects	**7**
Unit II	**Iterators and Modules** ▪ Python Iterators ▪ Python Polymorphism ▪ Python Scope ▪ Python Modules and Mathematical Functions ▪ Python String Formatting	**8**
Unit III	**Python Matplotlib** ▪ Matplotlib Pyplot, Plotting and Markers ▪ Matplotlib Line, Labels and Title ▪ Matplotlib Adding Grid Lines ▪ Matplotlib Subplot ▪ Matplotlib Scatter, Bars, Histograms and Pie Charts	**7**
Unit IV	**Some Applications:** ▪ Installation of NumPy and SciPy ▪ Applications of Python to matrices ▪ Applications of Python to groups ▪ Applications of Python to numerica analysis ▪ Applications of Python to differential equations	**8**
Study Resources	▪ Downey, A. (2015). *Think Python: How to Think Like a Computer Scientist* (2nd ed.). O'Reilly Media, Inc. ▪ Johansson, R. (2019). *Numerical Python: Scientific Computing and Data Science Applications with Numpy, SciPy and Matplotlib* (2nd ed.). Apress. ▪ Langtangen, H. P. (2009). *Python Scripting for Computational Science* (3rd ed.). Springer Berlin Heidelberg.	

T.Y. B.Sc. Mathematics (Vocational)
Semester-V
MTH-VSC-352: Practical on MTH-VSC-351

Total Hours: 60 **Credits: 2**

Course Objectives	▪ To understand conditional statements. ▪ To study iterators and modules. ▪ To know the plotting of various graphs and charts using Python Matplotlib. ▪ To know the applications of NumPy and SciPy for solving problems in the subjects of curriculum.	
Course Outcomes	After successful completion of this course, students are expected to: ▪ create and run Python programs using conditional statements. ▪ use iterators and modules for solving mathematical problems. ▪ apply Python Matplotlib to plot various graphs and charts. ▪ apply NumPy and SciPy for solving problems in the subjects of curriculum.	
Sr. No.	**Contents**	**Hours**
1	Conditional Statements	**4**
2	Chained and nested conditionals	**4**
3	Looping Statements	**4**
4	Python Functions	**4**
5	Python Iterators and Polymorphism	**4**
6	Python Scope and Modules	**4**
7	Mathematical Functions	**4**
8	Python String Formatting	**4**
9	Matplotlib Pyplot, Plotting and Markers	**4**
10	Matplotlib Line, Labels and Title	**4**
11	Matplotlib Scatter, Bars, Histograms and Pie Charts	**4**
12	Applications of Python to matrices	**4**
13	Applications of Python to groups	**4**
14	Applications of Python to numerica analysis	**4**
15	Applications of Python to differential equations	**4**
Study Resources	▪ Downey, A. (2015). *Think Python: How to Think Like a Computer Scientist* (2nd ed.). O'Reilly Media, Inc. ▪ Johansson, R. (2019). *Numerical Python: Scientific Computing and Data Science Applications with Numpy, SciPy and Matplotlib* (2nd ed.). Apress. ▪ Langtangen, H. P. (2009). *Python Scripting for Computational Science* (3rd ed.). Springer Berlin Heidelberg.	

T.Y. B.Sc. Mathematics (On Job Training)
Semester-V
MTH-OJT-351: On Job Training/Internship

Total Hours: 120 **Credits:4**

Course objectives	▪ To provide the students with actual work experience. ▪ To make aware prescribe standards and guidelines at work. ▪ To develop the employability of participating student. ▪ To avail an opportunities to eventually acquire job experiences.
Course outcomes	After successful completion of this course, students are expected to: ▪ Get actual work experience with office and virtual exposure to various management styles, technical, industrial, and proceduralsystems. ▪ Acquaint the knowledge related to working hours, work protocols and guidelines. ▪ Understand the roles and responsibilities of employee as well as team work. ▪ Justify job experiences that match their potentials, skills, and competencies.

Internship

An internship is a professional learning experience that offers meaningful, practical work related to a student's field of study or career interest. An internship gives a student the opportunity for career exploration and development, and to learn new skills.

On the job training

On the job training is a form of training provided at the workplace. During the training, employees are familiarized with the working environment they will become part of. Employees also get a hands-on experience using machinery, equipment, tools, materials, etc.

Internship / OJT Mechanism:

1. **Pre-Approval**: Students should seek approval from the college before starting the Internship / OJT. This ensures that the Internship / OJT aligns with the curriculum and meets the necessary criteria.
2. **Mentor and Supervisor**: Each student should have an assigned mentor at the organization/industry where they are interning. Additionally, an Internship / OJT supervisor from the college will be appointed to guide and monitor the progress.
3. **Regular Reporting:** Students should maintain regular communication with their supervisor and mentor, providing progress reports and seeking feedback.
4. **Professional Conduct**: Students must adhere to professional conduct throughout the Internship / OJT, including punctuality, respect for colleagues, and adherence to the organization's/industry’s policies and guidelines.
5. **Student Diary**: Students should maintain a diary to document their experiences, challenges faced, and lessons learned during the Internship / OJT.
6. **Final Report**: At the end of the Internship / OJT, students should submit a comprehensive final report, summarizing their accomplishments, contributions, and key takeaways.
7. **Evaluation**: The Internship / OJT is worth 4 credits (equivalent to 100 marks), and the evaluation will be divided into two categories: one by the mentor and the other by the Internship / OJT supervisor. The mentor's evaluation (internal examination) will carry 40 marks, and it will be based on the student's performance during the Internship / OJT. External examination will be conducted by mentor and supervisor which will be based on the student's diary, the final report prepared by the student, and their performance in the final viva voce, and will carry 60 marks. The total marks obtained by the students in both evaluations will be added together for the purpose of final evaluation. The evaluation of the students will be conducted by the mentor using the evaluation sheet provided by the college.

Internal Evaluation Criteria for Students by the Mentor:

1. **Quality of Work** (10 marks): How well did the student perform their assigned tasks during the Internship / OJT? Evaluate the accuracy, thoroughness, and attention to detail in their work.
2. **Initiative and Proactiveness** (10 marks): Did the student show initiative in taking on additional responsibilities or tasks beyond their assigned role? Did they demonstrate a proactive attitude towards problem-solving?
3. **Communication Skills** (10 marks): Assess the student's ability to communicate effectively with colleagues, superiors, and clients (if applicable). Consider both written and verbal communication.
4. **Problem-Solving Skills and Time Management** (10 marks): Evaluate the student's ability to analyze problems, propose solutions, and implement effective strategies to overcome challenges. How well did the student manage their time during the Internship / OJT? Were they able to meet project deadlines and handle multiple tasks efficiently?

External Evaluation Criteria for Students by the Supervisor and Mentor:

1. **Student Diary** (15 marks): Review the student's diary to understand their reflections, insights gained, and self-assessment of their performance during the Internship / OJT.
2. **Final Report** (15 marks): Evaluate the quality and comprehensiveness of the student's final report, including the clarity of their achievements and contributions.
3. **Presentation of Student in Viva Voce** (30 marks): Evaluate the responses given by the student to the questions asked by the faculty in the Viva Voce.

Evaluation Criteria for Final Viva Voce:

1. Presentation Skills
2. Knowledge of the Internship / OJT Project
3. Practical Application and Work Experience
4. Problem-Solving and Critical Thinking
5. Communication and Professionalism

SEMESTER-VI

T.Y. B.Sc. Mathematics (Major)
Semester-VI
MTH-DSC-361: Dynamical Systems

Total Hours: 30 **Credits: 2**

Course Objectives	▪ To study the concept of equilibrium points and stability of dynamical systems. ▪ To learn how to formulate mathematical models for real-world phenomena using dynamical systems (e.g., biological, mechanical, economic, etc.). ▪ To understand how to visualize and interpret the behavior of systems computationally. ▪ To introduce the main features of dynamical systems, particularly as they arise from systems of ordinary differential equations as models in applied mathematics.	
Course Outcomes	After successful completion of this course, students are expected to: ▪ analyze the stability of equilibrium points using method like linearization. ▪ formulate dynamical systems models for real-world phenomena from various fields, such as biology, physics, and economics. ▪ use computational tools to simulate and visualize the behavior of dynamical systems, understanding the qualitative nature of solutions. ▪ apply appropriate mathematical techniques, such as phase plane analysis, to study the qualitative behavior of dynamical systems.	
Unit	**Contents**	**Hours**
Unit I	**First-Order Equations** ▪ System of first order differential equations ▪ Solution graph and phase line ▪ The Logistic Population Model ▪ Bifurcations	**7**
Unit II	**Planar Linear Systems** ▪ Planar linear systems ▪ Eigenvalues and eigenvectors ▪ Solving linear systems ▪ The Linearity Principle	**8**
Unit III	**Phase Portraits** ▪ Saddle, source and sink ▪ Center, spiral sink and spiral source ▪ Repeated eigen values with single eigenvector ▪ Changing coordinates	**8**
Unit IV	**Classification of Planar Systems and Exponential of Matrix** ▪ The Trace-determinant plane, Dynamical classification ▪ Exponential of a matrix ▪ Solving a system of first order differential equations by using the exponential of a matrix.	**7**
Study Resources	▪ Morris Hirsch, S. Smale and Devaney, Differential Equations, Dynamical Systems and Introduction to Chaos, Academic Press, Elsevier, Second Edition, 2004. (Ch.-1: 1.1-1.6, Ch.-2: 2.1-2.7, Ch.-3: 3.1-3.4, Ch.-4: 4.1-4.2, Ch.-6: 6.4) ▪ Lawrence Perko, Differential Equations and Dynamical Systems, Springer-	

	Verlag, New York, Third Edition, 2001. (Ch.-1: 1.1-1.4)	

T.Y. B.Sc. Mathematics (Major)
Semester-VI
MTH-DSC-362: Linear Algebra

Total Hours: 30 **Credits: 2**

<table>
<tr><td>Course Objectives</td><td colspan="2">▪ To know vector spaces, subspaces, quotient space and linear span.
▪ To study basis and dimension of finite dimensional vector spaces.
▪ To study linear transformations.
▪ To know properties of eigenvalues, eigenvectors and diagonalisation of matrices.</td></tr>
<tr><td>Course Outcomes</td><td colspan="2">After successful completion of this course, students are expected to:
▪ understand subspaces, linear span and their properties which is one of the building blocks of pure mathematics.
▪ explain basis and dimensions, Extension theorem.
▪ explain concepts of linear transformations, singular, non-singular and invertible linear transformations.
▪ learn eigenvalues and eigenvectors, diagonalisation of matrices.</td></tr>
<tr><td>Unit</td><td>Contents</td><td>Hours</td></tr>
<tr><td>Unit I</td><td>Vector Spaces
▪ Vector spaces, Subspaces, Examples
▪ Necessary and sufficient conditions for a subspace
▪ Addition, Intersection and union of subspaces
▪ Quotient space, linear span and its properties</td><td>7</td></tr>
<tr><td>Unit II</td><td>Basis and Dimensions
▪ Linear dependence and independence
▪ Basis and dimension of finite dimensional vector spaces
▪ Extension theorem
▪ Theorems on basis and dimensions</td><td>8</td></tr>
<tr><td>Unit III</td><td>Linear Transformations
▪ Linear transformation and examples
▪ Kernel and image of linear transformations
▪ Isomorphism theorems and Rank-nullity theorem
▪ Algebra of linear transformations
▪ Singular and non-singular linear transformations
▪ Invertible linear transformations</td><td>7</td></tr>
<tr><td>Unit IV</td><td>Eigenvalues and Eigenvectors
▪ Matrix polynomial, Characteristics polynomial, Minimum polynomial
▪ Eigenvalues and Eigenvectors
▪ Cayley Hamilton theorem
▪ Matrix representation of linear operator and linear transformation
▪ Diagonalisation of Matrices</td><td>8</td></tr>
<tr><td>Study Resources</td><td>▪ N. S. Gopalkrishnan, University Algebra, New Age Int. Pvt. Ltd, 2015.
▪ S. Lipschutz, Theory and Problems of Linear Algebra, Schaum’s outline series, SI(Metric) edition, McGraw Hill Book Company, 1987.
▪ Vivek Sahai and Vikas Bist, Linear Algebra, Second Edition, Narosa Publishing house, 2022.
▪ Balaram Dubey, Introductory Linear Algebra, Asian books Int. Pvt. Ltd, 2007.</td><td></td></tr>
</table>

T.Y. B.Sc. Mathematics (Major)
Semester-VI
MTH-DSC-363: Metric Sapces

Total Hours: 30 **Credits: 2**

Course Objectives	▪ To know basics concepts of matric spaces. ▪ To study continuous functions on metric spaces. ▪ To learn connected and complete of metric spaces. ▪ To discuss compactness of matric spaces.	
Course Outcomes	After successful completion of this course, students are expected to: ▪ understand equivalence and countability of sets, basics of metric space and limits on it. ▪ test continuity and homeomorphism of a function on metric spaces. ▪ find the interrelation bnetween connected and complete of metric spaces. ▪ Check compactness of metric spaces.	
Unit	**Contents**	**Hours**
Unit I	**Metric Spaces** ▪ Equivalence and Countability ▪ Metric Spaces ▪ Limits in Metric Spaces	**7**
Unit II	**Continuous functions on Metric Spaces** ▪ Reformulation of definition of continuity in Metric Spaces. ▪ Continuous function on Metric Spaces. ▪ Open Sets ▪ Closed Sets ▪ Homeomorphisms.	**8**
Unit III	**Connected Metric Spaces and Complete of Metric Spaces** ▪ More about Sets ▪ Connected Set ▪ Bounded and Totally bounded sets ▪ Complete Metric Spaces ▪ Properties of Complete Metric Spaces ▪ Contraction Mapping on Metric Spaces.	**7**
Unit IV	**Compactness of Metric Spaces** ▪ Compact Metric Spaces. ▪ Continuous function on compact Metric Spaces. ▪ Continuity of inverse function ▪ Uniform Continuity	**8**
Study Resources	▪ R. R. Goldberg, *Methods of Real Analysis*, Oxford & IBH Publishing Co. PVT. LTD, 2nd Edition, 1976. Chapter I : 1.5 ,1.6 , Chapter IV : 4.2, 4.3, Chapter V : 5.2,5.3, 5.4, 5.5 Chapter VI : 6.1,6.2,6.3.6.4,6.5,6.6,6.7,6.8 ▪ S. C. Malik and SavitaArora, *Mathematical Analysis*, Second Edition, New Age International Pvt. Ltd., New Delhi, 2010. ▪ D. Somsundaram and B. Chaudhari, *A First Course in Mathematical Analysis*, Narosa Publishing House, New Delhi. 2018.	

T.Y. B.Sc. Mathematics (Major)
Semester-VI
MTH-DSC-364: Laplace Transforms

Total Hours: 30 **Credits: 2**

<table>
<tr><td>Course Objectives</td><td colspan="2">▪ To know the concept of Laplace transforms and its properties.
▪ To study the concept of inverse Laplace transforms and its properties.
▪ To understand the concept of convolution.
▪ To study methods in Laplace transforms which are useful for solving differential equations.</td></tr>
<tr><td>Course Outcomes</td><td colspan="2">After successful completion of this course, students are expected to:
▪ find Laplace transforms of given function.
▪ solve Inverse Laplace transforms of given function.
▪ apply convolution theorem to find integration.
▪ apply Laplace transform in solving differential equations.</td></tr>
<tr><td>Unit</td><td>Contents</td><td>Hours</td></tr>
<tr><td>Unit I</td><td>Laplace Transforms
▪ Definition and existence of Laplace transform.
▪ Laplace transforms of elementary functions and validity.
▪ Properties of Laplace transform.
▪ Laplace transforms of derivatives and real integrals.
▪ Multiplication by t^n.
▪ Division by t.</td><td>7</td></tr>
<tr><td>Unit II</td><td>Inverse Laplace Transform
▪ Definition and use of table.
▪ Properties of inverse Laplace transforms.
▪ Inverse Laplace transforms of derivatives and real integrals.
▪ Multiplications by s.
▪ Division by s.</td><td>8</td></tr>
<tr><td>Unit III</td><td>Convolution Theorem
▪ Laplace transforms of periodic functions.
▪ Convolution theorem.
▪ Evaluation of inverse Laplace transform by convolution theorem.
▪ Use of partial fractions.</td><td>7</td></tr>
<tr><td>Unit IV</td><td>Applications to Differential Equations
▪ Solution of linear differential equation with constant coefficients by using Laplace transforms.
▪ Laplace transforms of Heaviside's unit step functions.
▪ Laplace transforms of Dirac-delta functions.</td><td>8</td></tr>
<tr><td>Study Resources</td><td>▪ Murry R. Spiegel, Theory and problems of Laplace transforms, Schaum's Outline Series, McGraw Hill Ltd, New York, 1965.
▪ A. R. Vasishtha and R. K. Gupta, Integral transforms, Krishna Prakashan Media (P) Ltd.</td><td></td></tr>
</table>

T.Y. B.Sc. Mathematics (Major with IKS)
Semester-VI
MTH-DSC-365: Ancient Indian Mathematics

Total Hours: 30 **Credits: 2**

Course Objectives	▪ To Study the contributions of ancient Indian Mathematicians ▪ To study the multiplication of numbers by using Vedic Mathematics techniques. ▪ To study the division of numbers by using Vedic Mathematics techniques. ▪ To study the exponent of numbers by using Vedic Mathematics techniques.	
Course Outcomes	After successful completion of this course, students are expected to: ▪ familiar with the work of ancient Indian Mathematicians. ▪ knows and able to use the Vedic Mathematics techniques for multiplication of numbers. ▪ knows and able to use the Vedic Mathematics techniques for division of numbers. ▪ knows and able to use the Vedic Mathematics techniques for exponent of numbers.	
Unit	**Contents**	**Hours**
Unit I	**Work of Indian Mathematicians** ▪ Aryabhatt ▪ Brahmagupt ▪ Bhaskara I ▪ Mahaviracharya ▪ Madhava ▪ Srinivasa Ramanujan	**7**
Unit II	**Multiplication** ▪ Ekadhikenpurven method (multiplication of two numbers of two digits) ▪ Eknunenpurven method (multiplication of two numbers of three digits) ▪ Urdhvatiragbhyam method (multiplication of two numbers of three digits) ▪ NikhilamNavtashchramamDashtaha (multiplication of two numbers of three digits) ▪ Combined Operations	**8**
Unit III	**Division and Divisibility** ▪ **Division** o NikhilamNavtashchramamDashtaha (two digits divisor) o ParavartyaYojyet method (three digits divisor) ▪ **Divisibility** o Ekadhikenpurven method (two digits divisor) o Eknunenpurven method (two digits divisor)	**7**
Unit IV	**Power and Root** ▪ Square (two digit numbers) ▪ Cube (two digit numbers) ▪ Square root (four digit number) ▪ Cube root (six digit numbers)	**8**
Study Resources	▪ Jagadguru Swami Bharati Krishna Tirthaji Maharaja and V. S. Agarwala, Vedic Mathematics. Motilal Banarsidass, New Delhi, 1985. ▪ B.G. Bapat and Dilip Kulkarni, Vaidik Ganit (Part-1). Smita Printers, Pune, 1982.	

	▪ B.G. Bapat and Dilip Kulkarni, Vaidik Ganit (Part-2). Smita Printers, Pune, 1983. ▪ B.G. Bapat and Dilip Kulkarni, Vaidik Ganit (Part-3). Smita Printers, Pune, 1984. ▪ B.G. Bapat and Dilip Kulkarni, Vaidik Ganit (Part-4). Smita Printers, Pune, 1985. ▪ B. Mahadevan, Nagendra Pavana, and Vinayak Rajat Bhat, Introduction to Indian Knowledge System : Concepts and Applications. PHI Learning, Delhi, 2022.	

T.Y. B.Sc. Mathematics (Major)
Semester-VI
MTH-DSC-366: Practical on MTH-DSC-361

Total Hours: 60 **Credits: 2**

Course Objectives	▪ To study the concept of equilibrium points and stability of dynamical systems. ▪ To learn how to formulate mathematical models for real-world phenomena using dynamical systems (e.g., biological, mechanical, economic, etc.). ▪ To understand how to visualize and interpret the behavior of systems computationally. ▪ To introduce the main features of dynamical systems, particularly as they arise from systems of ordinary differential equations as models in applied mathematics.	
Course Outcomes	After successful completion of this course, students are expected to: ▪ analyze the stability of equilibrium points using method like linearization. ▪ formulate dynamical systems models for real-world phenomena from various fields, such as biology, physics, and economics. ▪ use computational tools to simulate and visualize the behavior of dynamical systems, understanding the qualitative nature of solutions. ▪ apply appropriate mathematical techniques, such as phase plane analysis, to study the qualitative behavior of dynamical systems.	
Sr. No.	**Contents**	**Hours**
1	First order equations	**4**
2	Qualitative behavior	**4**
3	Phase line	**4**
4	Bifurcation	**4**
5	Planar linear systems	**4**
6	Eigenvalues and eigenvectors	**4**
7	Solving linear systems-I	**4**
8	Solving linear systems-II	**4**
9	Phase Portraits-I	**4**
10	Phase Portraits-II	**4**
11	Phase Portraits-III	**4**
12	Changing coordinates	**4**
13	Trace-determinant plane	**4**
14	Exponential of a matrix-I	**4**
15	Exponential of a matrix-II	**4**
Study Resources	▪ Morris Hirsch, S. Smale and Devaney, Differential Equations, Dynamical Systems and Introduction to Chaos, Academic Press, Elsevier, Second Edition, 2004. ▪ Lawrence Perko, Differential Equations and Dynamical Systems, Springer-Verlag, New York, Third Edition, 2001.	

List of Practicals

Practical No. - 1: First order equations

1. Find the general solution of the differential equation $x' = ax + 3$, where a is a parameter.

2. Find the general solution of the differential equation $x' = ax(1-x)$, where a is a parameter.
3. For the differential equation $x' = \cos x$, find all equilibrium solutions and determine whether they are sorce, sink or neither.
4. For the differential equation $x' = x^2 - x - 2$, find all equilibrium solutions and determine whether they are sorce, sink or neither.
5. For the differential equation $x' = x^3 + x^2 - 2x$, find all equilibrium solutions and determine whether they are sorce, sink or neither.

Practical No. - 2: Qualitative behavior

1. Discuss the qualitative behavior of $x' = x^3 - x$.
2. Discuss the qualitative behavior of $x' = x - x^3$.
3. Sketch the solution graphs for $x' = |1 - x^2|$.
4. Draw the solution graphs for $x' = \cos x$.
5. Draw the solution graphs for $x' = x^2 - x^4$.

Practical No. - 3: Phase line

1. Sketch the phase line for $x' = x - x^3$.
2. Sketch the phase line for $x' = x(1-x)$.
3. Sketch the phase line for $x' = \sin x$.
4. Sketch the slope field for $x' = x^3 - x$.
5. Sketch the slope field for $x' = x^4 - x^2$.

Practical No. - 4: Bifurcation

1. Sketch the bifurcation diagram for $x' = x^2 - ax$, where a is a parameter.
2. Sketch the bifurcation diagram for $x' = ax - x^2$, where a is a parameter.
3. Sketch the bifurcation diagram for $x' = a - x^2$, where a is a parameter.
4. Sketch the bifurcation diagram for $x' = x(1-x) - a$, where a is a parameter.
5. Sketch the bifurcation diagram for $x' = \frac{x}{10}(10 - x) - a$, where a is a parameter.

Practical No. - 5: Planar linear systems

1. Consider the second order differential equation with constant coefficients $x'' + ax' + bx = 0$. Convert this equation into an equivalent system of first order equation.
2. Convert the LCR circuit equation $LCx'' + RCx' + x = v(t)$ into an equivalent system of first order equation.
3. Consider the system $x' = x, y' = y$. Sketch the vector field.
4. Consider the system $x' = x, y' = -y$. Sketch the solution curves.
5. Consider the system $x' = -x, y' = y$. Sketch the vector field and solution curves.

Practical No. - 6: Eigenvalues and eigenvectors

1. Find the eigenvalues of the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix}$.
2. Describe all possible 2×2 matrices whose eigenvalues are 0 and 1.
3. If A is a non-singular matrix, then show that the eigenvalues of A^{-1} are the reciprocals of the eigenvalues of A.
4. Find the eigenvectors of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.
5. Find the eigenvectors of the matrix $A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$.

Practical No. - 7: Solving linear systems-I

1. Find two solutions of the system $\frac{dx}{dt} = x + 3y, \frac{dy}{dt} = x - y$.
2. Find two solutions of the system $\frac{dx}{dt} = -8x - 5y, \frac{dy}{dt} = 10x + +7y$.
3. Find the general solution of the differential eauation $x'' + 3x' + 2x = 0$.
4. Find the general solution of the differential eauation $x'' - 5x' + 6x = 0$.
5. Find the general solution of $x' = -x, y' = 2y$.

Practical No. - 8: Solving linear systems-II

1. Find the general solution of $X' = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} X$.
2. Find the general solution of $X' = \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix} X$.
3. Find the general solution of $X' = \begin{pmatrix} a & b \\ c & a \end{pmatrix} X$, where $bc > 0$.
4. Find the general solution of $X' = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} X$.

Practical No. - 9: Phase Portraits-I

1. Sketch the phase portrait of $X' = \begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix} X$.
2. Sketch the phase portrait of $X' = \begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix} X$.
3. Sketch the phase portrait of $X' = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} X$.
4. Sketch the phase portrait of $X' = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} X$.
5. Sketch the phase portrait of $X' = \begin{pmatrix} 14 & -10 \\ 5 & -1 \end{pmatrix} X$.

Practical No. - 10: Phase Portraits-II

1. Sketch the phase portrait of $X' = \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix} X$.
2. Sketch the phase portrait of $X' = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} X$.
3. Sketch the phase portrait of $X' = \begin{pmatrix} -1 & 3 \\ -3 & -1 \end{pmatrix} X$.
4. Sketch the phase portrait of $X' = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} X$.
5. Sketch the phase portrait of $X' = \begin{pmatrix} -2 & 1 \\ 0 & -2 \end{pmatrix} X$.

Practical No. - 11: Changing coordinates-I

1. Consider the system $X' = \begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix} X$. Find the matrix T that puts A in canonical form.
2. Consider the system $X' = \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix} X$. Find the matrix T that puts A in canonical form.
3. Consider the system $X' = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} X$. Find the matrix T that puts A in canonical form.
4. Consider the system $X' = \begin{bmatrix} 3 & 4 \\ -2 & -1 \end{bmatrix} X$. Sketch the phase portrait of $Y' = (T^{-1}AT)Y$, where $T^{-1}AT$ is in canonical form.
5. Consider the system $X' = \begin{bmatrix} -6 & 4 \\ -4 & 2 \end{bmatrix} X$. Sketch the phase portrait of $Y' = (T^{-1}AT)Y$, where $T^{-1}AT$ is in canonical form.

Practical No. - 12: Changing coordinates-II

1. Find the general solution of $X' = AX$ by obtaining canonical form of $A = \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}$.

2. Find the general solution of $X' = AX$ by obtaining canonical form of $A = \begin{bmatrix} -6 & 4 \\ -4 & 2 \end{bmatrix}$.
3. Find the general solution of $X' = AX$ by obtaining canonical form of $A = \begin{bmatrix} 3 & 4 \\ -2 & -1 \end{bmatrix}$.
4. Find the general solution of $X' = AX$ by obtaining canonical form of $A = \begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix}$.
5. Find the general solution of $X' = AX$ by obtaining canonical form of $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$.

Practical No. - 13: Trace-determinant plane

1. Classify the equilibrium points of the system $X' = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} X$.
2. Classify the equilibrium points of the system $X' = \begin{bmatrix} 4 & 2 \\ 3 & 2 \end{bmatrix} X$.
3. Classify the equilibrium points of the system $X' = \begin{bmatrix} -3 & -8 \\ 4 & -6 \end{bmatrix} X$.
4. Without finding the eigenvalue of the matrix $A = \begin{bmatrix} 1 & 2 \\ 7 & 4 \end{bmatrix}$, determine the nature of the eigenvalues.
5. Without finding the eigenvalue of the matrix $A = \begin{bmatrix} 5 & -3 \\ -8 & -6 \end{bmatrix}$, determine the nature of the eigenvalues.

Practical No. - 14: Exponential of a matrix-I

1. Find the exponentials of $A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$.
2. Find e^A, where $A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$.
3. Find e^{tA}, where $A = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$ and $\lambda \neq 0$.
4. Compute the exponentials of $A = \begin{bmatrix} 5 & -6 \\ 3 & -4 \end{bmatrix}$.
5. Show that if X is an eigenvector of A corresponding to the eigenvalue λ, then X is also an eigenvector of e^A corresponding to the eigenvalue e^λ.

Practical No. - 15: Exponential of a matrix-II

1. Find 2×2 matrices A and B such that $e^{A+B} \neq e^A e^B$.
2. Find the general solution of the system $X' = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$.
3. Solve the system $X' = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$.
4. Find e^{At} and solve the linear system $X' = \begin{pmatrix} -2 & -1 \\ 1 & -3 \end{pmatrix} X$ with $X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.
5. Solve the system $y_1' = 5y_1 + 4y_2, y_2' = y_1 + 2y_2$ subject to $y_1(0) = 2, y_2(0) = 3$.

T.Y. B.Sc. Mathematics (Major)
Semester-VI
MTH-DSC-367: Practical on MTH-DSC-362

Total Hours: 60 **Credits: 2**

Course Objectives	▪ To know vector spaces, subspaces, quotient space and linear span. ▪ To study basis and dimension of finite dimensional vector spaces. ▪ To study linear transformations. ▪ To know properties of eigenvalues, eigenvectors and diagonalisation of matrices.	
Course Outcomes	After successful completion of this course, students are expected to: ▪ understand subspaces, linear span and their properties which is one of the building blocks of pure mathematics. ▪ explain basis and dimensions, Extension theorem. ▪ explain concepts of linear transformations, singular, non-singular and invertible linear transformations. ▪ learn eigenvalues and eigenvectors, diagonalisation of matrices.	
Sr. No.	**Contents**	**Hours**
1	Vector spaces	**4**
2	Subspaces	**4**
3	Quotient space and Linear span	**4**
4	Linearly dependent sets	**4**
5	Linearly independent sets	**4**
6	Basis of vector spaces	**4**
7	Dimension of vector spaces	**4**
8	Linear Transformations	**4**
9	Kernel and Image of linear Transformation	**4**
10	Singular, Non-singular and Invertible linear transformations	**4**
11	Matrix of associate to a linear transformations	**4**
12	Eigenvalues	**4**
13	Eigenvectors	**4**
14	Cayley-Hamilton theorem	**4**
15	Similar and Diagonalizable Matrices	**4**
Study Resources	▪ N. S. Gopalkrishnan, *University Algebra*, New Age Int. Pvt. Ltd, 2015. ▪ S. Lipschutz, *Theory and Problems of Linear Algebra*, Schaum's outline series, SI(Metric) edition, McGraw Hill Book Company, 1987. ▪ Vivek Sahai and Vikas Bist, *Linear Algebra*, Second Edition, Narosa Publishing house, 2022. ▪ Balaram Dubey, *Introductory Linear Algebra*, Asian books Int. Pvt. Ltd, 2007.	

List of Practicals

Practical No. 1: Vector spaces

1. Is $\mathbb{R}$ a $\mathbb{C}$-vector space under usual addition and multiplication of numbers? Justify.

2. If $\mathbb{R}$ is the field reals and $\mathbb{R}^n = \{(x_1, x_2, x_3, \cdots, x_n): x_i \in \mathbb{R}$ for all $i\}$, then $\mathbb{R}^n$ is a $\mathbb{R}$-vector space under the following addition and scalar multiplication:
$$(x_1, x_2, x_3, \cdots, x_n) + (y_1, y_2, y_3, \cdots, y_n) = (x_1 + y_1, x_2 + y_2, x_3 + y_3, \cdots, x_n + y_n)$$
$$\lambda(x_1, x_2, x_3, \cdots, x_n) = (\lambda x_1, \lambda x_2, \lambda x_3, \cdots, \lambda x_n)$$
where $(x_1, x_2, x_3, \cdots, x_n), (y_1, y_2, y_3, \cdots, y_n) \in \mathbb{R}^n$ and $\lambda \in \mathbb{R}$.
3. Let F be a field and $F_{m\times n} = \{[\alpha_{ij}]_{m\times n}: \alpha_{ij} \in F$ for all $i, j\}$. Show that $F_{m\times n}$ is a F-vector space under the following addition and scalar multiplication:
$$A + B = [\alpha_{ij} + \beta_{ij}]_{m\times n} \text{ and } \lambda A = [\lambda \alpha_{ij}]_{m\times n}$$
for all $A = [\alpha_{ij}]_{m\times n}, B = [\beta_{ij}]_{m\times n} \in F_{m\times n}$ and $\lambda \in F$.
4. Consider an abelian group $V = \mathbb{R}^+$ under the usual multiplication of reals. Show that V is a $\mathbb{R}$-vector space under the scalar multiplication: $\alpha x = x^\alpha$ for all $x \in V$ and $\alpha \in \mathbb{R}$.
5. Is $V = \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbb{R}_{2\times 2}: ad - bc = 1\right\}$ a $\mathbb{R}$-vector space under usual addition and scalar multiplication of matrices.

Practical No. 2: Subspaces

1. Let F be a field and F^n a F-vector space. Denote $W = \{(a_1, a_2, a_3, \cdots, a_n) \in F^n: a_1 + a_2 + a_3 + \cdots + a_n = 0\}$. Show that W is a subspace of F^n.
2. Consider $\mathbb{R}^2$ as a $\mathbb{R}$-vector space and $W = \{(a, b) \in \mathbb{R}^2: a > 0\}$. Is W a subspace of $\mathbb{R}^2$? Justify.
3. Let $V = \mathbb{R}^3$ be a $\mathbb{R}$-vector space and $W = \{(x, y, z) \mid x, y, z \in \mathbb{R}$ and $x - 3y + 4z = 0\}$. Show that W is a subspace of V.
4. Let $V = \mathbb{R}^3$ be a $\mathbb{R}$-vector space and $W_1 = \{(a, b, 0) \in V : a, b \in \mathbb{R}\}$, $W_2 = \{(0, 0, c) \in V : c \in \mathbb{R}\}$ be subspaces of V. Is $W_1 \cup W_2$ a subspace of V? Justify.
5. Consider $\mathbb{R}^3$ as $\mathbb{R}$-vector space and $W_1 = \{(a, b, 0): a, b \in \mathbb{R}\}$ and $W_2 = \{(0, 0, c): c \in \mathbb{R}\}$. Find $W_1 \oplus W_2$.

Practical No. 3: Quotient space and Linear span

1. Let W be a subspace of a F-vector space V. Prove that $\bar{T}$ is a subspace of $\frac{V}{W}$ if and only if there exists a subspace V_1 of V such that $W \subseteq V_1$ and $\bar{T} = \frac{V_1}{W}$.
2. Consider $\mathbb{R}^3$ as a $\mathbb{R}$-vector space. Find the subspace generated by $(1, 2, 0)$ and $(0, 0, 4) \in \mathbb{R}^3$.
3. Consider $\mathbb{R}^3$ as a $\mathbb{R}$-vector space and $S = \{(1, 2, 1), (1, 1, -1), (4, 5, -2)\}$. Determine whether the vector $(1, 1, 0)$ is present in $L(S)$ or not.
4. Find the generating set for the subspace W of symmetric matrices in the $\mathbb{R}$-vector space $\mathbb{R}_{2\times 2}$.
5. Give an example of a F-vector space V and $S, T \subseteq V$ such that $L(S) \subseteq L(T)$ but $S \nsubseteq T$.

Practical No. 4: Linearly dependent sets

1. Consider $\mathbb{R}^3$ as a $\mathbb{R}$-vector space and $v_1 = (1, 0, 0), v_2 = (0, 1, 1), v_3 = (1, 1, 1) \in \mathbb{R}^3$. Show that $S = \{v_1, v_2, v_3\}$ is a linearly dependent set.
2. Let V be a F-vector space and $u, v, w \in V$. If $\{u, v, w\}$ is a linearly independent set, then prove that $\{u + v, v + w, w - u\}$ is a linearly dependent set.
3. Let V be a F-vector space and $u, v, w \in V$. If $\{u, v, w\}$ is a linearly independent set, then show that $\{u - v, v - w, w - u\}$ is a linearly dependent set.
4. In a $\mathbb{R}$-vector space $\mathbb{R}^3$, check whether the set $\{(3, 3, 1), (1, 1, 0), (0, 0, 1)\}$ is linearly dependent or not.
5. Consider $\mathbb{R}^3$ as a $\mathbb{R}$-vector space and $v_1 = (1, 0, 2), v_2 = (0, -2, 5), v_3 = (2, -6, 19) \in \mathbb{R}^3$. Show that $S = \{v_1, v_2, v_3\}$ is a linearly dependent set.

Practical No. 5: Linearly independent sets

1. Show that $\{1, i\}$ is a linearly dependent set in a $\mathbb{C}$-vector space $\mathbb{C}$ but it is linearly independent in a $\mathbb{R}$-vector space $\mathbb{C}$.
2. Let V be a F-vector space and $u, v, w \in V$. If $\{u, v, w\}$ is a linearly independent set, then prove that $\{u + v, v + w, w + u\}$ is a linearly independent set.
3. Let V be a F-vector space and $u, v, w \in V$. If $\{u, v, w\}$ is a linearly independent set, then prove that $\{u + v, u - v, u - 2v + w\}$ is a linearly independent set.
4. Consider $\mathbb{R}^4$ as a $\mathbb{R}$-vector space and $v_1 = (1, 0, 1, 0), v_2 = (0, 1, 0, 1),\ v_3 = (0, 0, 0, 2) \in \mathbb{R}^4$. Show that $S = \{v_1, v_2, v_3\}$ is a linearly independent set.
5. Consider $V = \{A \in \mathbb{R}_{3\times3}: A$ is an upper triangular matrix$\}$ as a $\mathbb{R}$-vector space. Find any linearly independent set in V containing six elements.

Practical No. 6: Basis of vector spaces

1. Consider $\mathbb{R}^n$ as a $\mathbb{R}$-vector space and $e_i = (0, 0, \cdots 0, 1, 0, \cdots, 0) \in \mathbb{R}^n$ for all $1 \leq i \leq n$ where 1 is at i^{th}-place. Show that $\{e_1, e_2, \cdots, e_n\}$ is a basis for $\mathbb{R}^n$ over $\mathbb{R}$.
2. Consider $\mathbb{R}^3$ as a $\mathbb{R}$-vector space and $v_1 = (1, 1,1), v_2 = (0, 1, 1),\ v_3 = (0, 0, 1) \in \mathbb{R}^3$. Show that $S = \{v_1, v_2, v_3\}$ is a basis for $\mathbb{R}^3$ over $\mathbb{R}$.
3. How many bases exists for a $\mathbb{C}$-vector space $\mathbb{C}$?
4. Consider a $\mathbb{R}$-vector space $P_3 = \{f(x) \in \mathbb{R}[x]: \deg f(x) \leq 3\} \cup \{0\}$ and $v_1 = 1, v_2 = 1 + x,\ v_3 = 1 + x^2, v_4 = 1 + x^3 \in P_3$. Show that $S = \{v_1, v_2, v_3\}$ is a basis for P_3 over $\mathbb{R}$.
5. Let $\mathbb{R}^3$ be a $\mathbb{R}$-vector space. Consider a basis $\{v_1 = (1,1,1), v_2 = (0,1,1), v_3 = (0,0,1)\}$. Let $v = (3,1,-4) \in \mathbb{R}^3$. Find the co-ordinate vector of v relative to basis $\{v_1, v_2, v_3\}$.

Practical No. 7: Dimension of vector spaces

1. Find the dimension of the $\mathbb{R}$-vector space $\mathbb{R}_{3\times3}$.
2. Consider $V = \{A \in \mathbb{R}_{3\times3}: A$ is an upper triangular matrix$\}$ as a $\mathbb{R}$-vector space. Find $\dim_{\mathbb{R}} V$.
3. With usual notations, find $\dim_{\mathbb{R}}(P_3)$.
4. Let W_1, W_2 be subspaces of a F-vector space V. If $dim(V) = 8, dim(W_1) = 6,\ dim(W_2) = 5$, then find possible dimensions of $W_1 \cap W_2$.
5. Consider $V = \{A: A$ is a symmetric matrix in $\mathbb{R}_{2\times2}\}$ as $\mathbb{R}$-vector space. Find $\dim_{\mathbb{R}}(V)$.

Practical No. 8: Linear Transformations

1. Let $\mathbb{R}^3,\ \mathbb{R}^2$ be $\mathbb{R}$-vector spaces. Define $T: \mathbb{R}^3 \to \mathbb{R}^2$ by $T(a, b, c) = (a, b)$ for all $(a, b, c) \in \mathbb{R}^3$. Show that T is a linear transformation.
2. Consider $\mathbb{R}[x]$ as a $\mathbb{R}$-vector space and $P_n = \{f(x) \in \mathbb{R}[x]: \deg f(x) \leq n\} \cup \{0\}$ where $n \in \mathbb{N}$. Show that the differential map $T: P_n \to P_n$ defined by $T(f(x)) = f'(x)$ is a linear transformation.
3. Find $T(a, b, c)$ where $T: \mathbb{R}^3 \to \mathbb{R}$ is a linear transformation defined by $T(1, 1, 1) = 3,\ T(0, 1, -2) = 1,\ T(0, 0, 1) = -2$.
4. Let $\mathbb{R}^3$ be a $\mathbb{R}$-vector space. Is $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(a, b, c) = (3a, b, c + 1)$ a linear transformation?
5. Let $\mathbb{R}^2, \mathbb{C}$ be $\mathbb{R}$-vector spaces. Define $T: \mathbb{R}^2 \to \mathbb{C}$ by $T(a, b) = a + ib$, for all $(a, b) \in \mathbb{R}^2$. Show that T is a linear transformation.

Practical No. 9: Kernel and Image of linear Transformation

1. Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation defined by $T(x, y) = (x, x + y, y)$. Find the range, rank, kernel and nullity of T.
2. Let $\mathbb{R}^2, \mathbb{R}^3$ be $\mathbb{R}$-vector spaces. Is a linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(x_1, x_2) = (x_1, x_1 - x_2, x_2)$ for all $(x_1, x_2) \in \mathbb{R}^2$ one-one?
3. Find a linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ whose image is generated by $(1, 2, 3)$ and $(4, 5, 6)$.
4. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by $T(1, 0, 0) = (1, 0, 1), T(0, 1, 0) = (0, 0, 1)$ and $T(0, 0, 1) = (1, 0, 0)$. Find $r(T)$.
5. Let V be a F-vector space and $\dim(V) = n$. If $ImT = kerT$, then show that n is even.

Practical No. 10: Singular, Non-singular and Invertible linear transformations

1. Show that the linear operator T on $\mathbb{R}^3$ defined by $T(a, b, c) = (a + b + c,\ b + c,\ c)$ is non-singular.
2. Let the linear operator on $\mathbb{R}^3$ be defined by $T(x, y, z) = (x + 2y, y - z, x + 2z)$. Is T non-singular? Justify.
3. Let T be the linear operator on $\mathbb{R}^3$ defined by $T(x, y, z) = (2x,\ 4x - y, 2x + 3y - z)$ for all $(x, y, z) \in \mathbb{R}^3$. Show that T is invertible and find a formula for T^{-1}.
4. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (z, x, y)$ for all $(x, y, z) \in \mathbb{R}^3$ be the linear transformation. Show that $T^3 = I$ and hence find rule of T^{-1}.
5. Show that the linear operator $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (-3x, -3y)$ for all $(x, y) \in \mathbb{R}^2$, is invertible.

Practical No. 11: Matrix of associate to a linear transformations

1. Let V be a n-dimensional F-vector space and $I: V \to V$ be a linear operator, then show that $m(I)$ with respect to any basis B is the identity matrix.
2. Consider $\mathbb{R}$-vector spaces $\mathbb{R}^3$ and $\mathbb{R}^2$. Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ be a linear operator defined by $T(x, y, z) = (x - y, y + z)$ for all $(x, y, z) \in \mathbb{R}^3$. If $B_1 = \{(1, 1, 1), (0, 1, 0), (1, 0, 0)\}$ is a basis for $\mathbb{R}^3$ over $\mathbb{R}$ and $B_2 = \{(0, 1), (1, 1)\}$ is a basis for $\mathbb{R}^2$ over $\mathbb{R}$, then matrix of T with respect find to basis B_1 and B_2.
3. Consider $\mathbb{R}$-vector space $P_3 = \{f(x) \in \mathbb{R}[x]: \deg f(x) \le 3\} \cup \{0\}$ and a linear operator $D: P_3 \to P_3$ defined by $D\big(f(x)\big) = \frac{d}{dx}\big(f(x)\big)$ for all $f(x) \in P_3$. Find the matrix of D with respect to the basis $B = \{1,\ x,\ x^2,\ x^3\}$.
4. If $m(T) = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix} \in \mathbb{R}_{3\times 3}$ is the matrix of the linear operator T on $\mathbb{R}^3$ with respect to the standard basis, then find the matrix of $T, m_1(T)$ with respect to the basis $B_1 = \{(1,1,1), (1, 1, 0), (1, 0, 0)\}$.
5. Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \in \mathbb{R}_{3\times 3}$. Find a linear operator $T: \mathbb{R}^3 \to \mathbb{R}^3$ such that the matrix of T with respect to the standard basis is A.

Practical No. 12: Eigenvalues

1. Find the characteristic equation for the matrice $A = \begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}$.
2. Let A be a square matrix. Show that A and A^t have same eigenvalues.
3. If the product of two eigenvalues of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16, then find the third eigenvalue of A.

4. Find the eigenvalues of the matrix $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$.
5. Let A be a 3×3 real matrix such that $|A| = 6$ and the $tr(A) = 0$. If $|A + I| = 0$, then compute all the eigenvalues of A.

Practical No. 13: Eigenvectors

1. Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$.
2. Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$.
3. Find the eigenvalues of $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$. Find the eigen space corresponding to eigenvalue $\lambda = 1$.
4. Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 6 \end{bmatrix}$.
5. Find eigenvalues and eigenvectors of the matrix $= \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$.

Practical No. 14: Cayley-Hamilton theorem

1. Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & -5 \\ 3 & 2 \end{bmatrix}$.
2. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$. Find A^{-1} by using Cayley-Hamilton Theorem.
3. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$, then using Cayley-Hamilton Theorem to find $A^2 - A$.
4. Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$.
5. Verify Cayley-Hamilton Theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and also find A^{-1}.

Practical No. 15: Similar and Diagonalizable Matrices

1. Find minimum polynomial of $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$.
2. Show that $A = \begin{bmatrix} 1 & -2 \\ 4 & 0 \end{bmatrix}$ and $A = \begin{bmatrix} 12 & 7 \\ -20 & -11 \end{bmatrix}$ are similar matrices.
3. If A, B are diagonalizable matrices with same minimum polynomial, then show that A and B are similar.
4. Show t hat $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$ is a diagonalizable matrix.
5. Show that $A = \begin{bmatrix} -3 & 1 & -3 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{bmatrix}$ is not a diagonalizable matrix.

T.Y. B.Sc. Mathematics (Elective)
Semester-VI
MTH-DSE-361A: Partial Differential Equations

Total Hours: 30 **Credits: 2**

Course Objectives	▪ To study the basics of a partial differential equation. ▪ To understand methods for solving the linear and non-linear partial differential equations. ▪ To study compatible systems of a partial differential equation. ▪ To learn general methods of solving partial differential equations.	
Course Outcomes	After successful completion of this course, students are expected to: ▪ find the order and degree of partial differential equation, ▪ solve the linear and non-linear partial differential equations. ▪ identify compatiblity of a partial differential equation. ▪ use Charpit's and Jacobi's methods to solve partial differential equations.	
Unit	**Contents**	**Hours**
Unit I	**Origin of Partial Differential Equations** ▪ Order and degree of partial differential equation ▪ Linear and non-linear partial differential equation ▪ Classification of first order partial differential equations ▪ Origin of partial differential equations ▪ Derivation of partial differential equation by elimination of arbitrary constants and arbitrary functions	**8**
Unit II	**Linear and Non-linear Partial Differential Equations** ▪ Lagrange's equations and Lagrange's method of solving $Pp + Qq = R$ ▪ Integral surface passing through a given curve ▪ Surfaces orthogonal to a given system of surfaces ▪ Complete integral (or complete solution), particular integral, singular integral (or singular solution) and general integral (or general solution) ▪ Geometrical interpretation of integrals of $f(x, y, z, p, q) = 0$	**8**
Unit III	**Compatible Systems** ▪ Method of getting singular integral directly from the partial differential equation of first order ▪ Compatible system of first-order equations ▪ Condition for system of two first order partial differential equation to be compatible ▪ A particular case of Compatible System	**7**
Unit IV	**General Methods of Solving Partial Differential Equations** ▪ Charpit's method and examples ▪ Special type (a) $f(p, q) = 0$, (b)$f(z, p, q) = 0$, (c) $g(x, p) = f(y, q)$ ▪ Examples on type (a), (b) and (c). ▪ Jacobi's method and examples	**7**
Study Resources	▪ M. D. Raisinghania, *Ordinary and Partial Differential Equations*, S. Chand and Company Pvt Ltd, Nineteenth Edition, 2017. (Part-III: 1.1-1.12, 2.1-2.2, 2.14-2.17, 3.1-3.10, 3.14-3.22, 9.1-9.4) ▪ T. Amaranath, *An Elementary Course in Partial Differential Equations*, Alpha Science International Ltd, Second Edition, 2003. ▪ Ian N. Sneddon, *Elements of Partial Differential Equations*, McGraw-Hill, Dover Edition, 2006.	

T.Y. B.Sc. Mathematics (Elective)
Semester-VI
MTH-DSE-361B: Computational Mathematics with SageMath
(NPTEL course code: noc25-ma16)

Total Hours: 30 **Credits: 2**

Course Objectives	▪ To learn SageMath and its basic applications to mathematics. ▪ To study calculus of one variable with SageMath. ▪ To study integration with SageMath. ▪ To study limits, continuity and partial derivatives with SageMath.	
Course Outcomes	After successful completion of this course, students are expected to: ▪ able to solve equations, 2d plot and 3d plot by using SageMath. ▪ able to use SageMath for calculus of one variable. ▪ able to find integration by using SageMath. ▪ able to calculate local maximum and local minimum by using SageMath.	
Unit	**Contents**	**Hours**
Unit I	**Introduction to SageMath** ▪ Introduction and Installation of SageMath ▪ Exploring integers in SageMath ▪ Solving Equations in SageMath ▪ 2d Plotting with SageMath ▪ 3d Plotting with SageMath	**7**
Unit II	**Calculus of one variable with SageMath** ▪ Calculus of one variable with SageMath -I ▪ Calculus of one variable with SageMath - II ▪ Applications of derivatives using SageMath	**8**
Unit III	**Integration with SageMath** ▪ Integration with SageMath ▪ Improper Integral using SageMath ▪ Application of integration using SageMath	**7**
Unit IV	**Advance calculus with SageMath** ▪ Limit and Continuity of real valued functions ▪ Partial Derivative with SageMath ▪ Local Maximum and Minimum ▪ Application of local maximum and local minimum	**8**
Study Resources	▪ www.sagemath.org ▪ Mathematical Computation with Sage by Paul Zimmermann available from on http://www.sagemath.org ▪ A First Course in Linear Algebra by Robert Beezer available online http://linear.ups.edu/ ▪ Abstract Algebra: Theory and Applications by Tom Judson and Robert Beezer (http://abstract.ups.edu/) ▪ An Introduction to SAGE Programming: With Applications to SAGE Interacts for Numerical Methods by Razvan A Mezei, Springer	

T.Y. B.Sc. Mathematics (Elective)
Semester-VI
MTH-DSE-362A: Practical on MTH-DSE-361A

Total Hours: 60 **Credits: 2**

Course Objectives	▪ To study the basics of a partial differential equation. ▪ To understand methods for solving the linear and non-linear partial differential equations. ▪ To study compatible systems of a partial differential equation. ▪ To learn general methods of solving partial differential equations.	
Course Outcomes	After successful completion of this course, students are expected to: ▪ find the order and degree of partial differential equation, ▪ solve the linear and non-linear partial differential equations. ▪ identify compatiblity of a partial differential equation. ▪ use Charpit's and Jacobi's methods to solve partial differential equations.	
Sr. No.	**Contents**	**Hours**
1	Origin of partial differential equations-I	**4**
2	Origin of partial differential equations-II	**4**
3	Origin of partial differential equations-III	**4**
4	Origin of partial differential equations-IV	**4**
5	Lagrange's Method-I	**4**
6	Lagrange's Method-II	**4**
7	Lagrange's Method-III	**4**
8	Integral Surface	**4**
9	Compatible systems-I	**4**
10	Compatible systems-II	**4**
11	Charpit's Method-I	**4**
12	Charpit's Method-II	**4**
13	Jacobi's Method-I	**4**
14	Jacobi's Method-II	**4**
15		**4**
Study Resources	▪ M. D. Raisinghania, *Ordinary and Partial Differential Equations*, S. Chand and Company Pvt Ltd, Nineteenth Edition, 2017. (Part-III: 1.1-1.12, 2.1-2.2, 2.14-2.17, 3.1-3.10, 3.14-3.22, 9.1-9.4) ▪ T. Amaranath, *An Elementary Course in Partial Differential Equations*, Alpha Science International Ltd, Second Edition, 2003. ▪ Ian N. Sneddon, *Elements of Partial Differential Equations*, McGraw-Hill, Dover Edition, 2006.	

List of Practicals

Practical No. - 1: Origin of partial differential equations-I

1. For each of the following partial differential equations, classify as linear or non-linear and find its order and degree:
   a. $\frac{\partial^2 z}{\partial x^2} = (1 + \frac{\partial z}{\partial y})^{\frac{1}{2}}$
   b. $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = xyz$

c. $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=z+xy$

2. Find a partial differential equation by eliminating a and b from $z=ax+by+a^2+b^2$.
3. Eliminate arbitory constant a and b from $z=(x-a)^2+(y-b)^2$.
4. Eliminate arbitory constant a and b from $z=a(x+y)+b$.
5. Eliminate arbitory constant a and b from $z=ax+by+ab$.

Practical No. - 2: Origin of partial differential equations-II

1. From differential equation by eliminating constant A and P from $z=Ae^{pt}sinpx$
2. Find a partial differential equation by eliminating a and b from $2z=(ax+y)^2+b$.
3. Find the differential equation of all sphere of radius λ,having centre in xy-plane.
4. Eliminate a,b,and c from $z=a(x+y)+b(x-y)abt+c$.
5. Find differential equation of the set of all right circular cones whoes whoes axes coincide with z-axis.

Practical No. - 3: Origin of partial differential equations-III

1. Form a partial differential equation by eliminating the arbitory function $\emptyset$ from $\emptyset(x+y+z,x^2+y^2-z^2)=0$.what is the order of this partial differential equation?
2. Form a partial differential equation by eliminating the arbitory function f from the equation $x+y+z=f(x^2+y^2+z^2)$.
3. Eliminate the arbitrary function f and F from $y=f(x-at)+F(x+at)$.
4. Eliminate the arbitrary function f from $z=f(x^2-y^2)$.
5. Form a partial differential equation by eliminating the arbitory function f from $z=f(\frac{y}{x})$.

Practical No. - 4: Origin of partial differential equations-IV

1. Form a partial differential equation by eliminating the arbitory function f from$z=x^nf(\frac{y}{x})$.
2. Form a partial differential equation by eliminating the arbitory function ϕ from $lx+my+nz=\phi(x^2+y^2+z^2)$.
3. Form a partial differential equation by eliminating the arbitory function f from $z=e^{ax+by}f(ax-by)$.
4. Form a partial differential equation by eliminating the arbitory function f and I from $z=f(x+iy)+F(x-iy)$.
5. Form a partial differential equation by eliminating the arbitory function f and g from $z=f(x^2-y)+g(x^2+y)$

Practical No. - 5: Lagrange's Method-I

1. Solve $ptanx+qtany=tanz$.
2. Solve $y^2p-xyq=x(z-2y)$.
3. Solve $p+3q=5z+\tan(y-3x)$.
4. Solve $z(z^2+xy)(px-qy)=x^4$.
5. Solve $xyp+y^2q=zxy-2x^2$.

Practical No. - 6: Lagrange's Method-II

1. Solve $xzp+yzq=xy$
2. Solve $py+qx=xyz^2(x^2-y^2)$.
3. Solve $xp-yq=xy$.
4. Solve $z(x+y)p+z(x-y)q=x^2+y^2$.
5. Solve $x(y^2+z)p-y(x^2+z)q=z(x^2-y^2)$

Practical No. - 7: Lagrange's Method-III

1. Solve $(y+z)p+(z+x)q=x+y$.
2. Solve $y^2(x-y)p+x^2(y-x)q=z(x^2+y^2)$.
3. Solve $(x^2-y^2-z^2)p+2xyq=2xz$.
4. Find the general integral of $xzp+yzq=xy$.

5. Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$.

Practical No. - 8: Integral Surface

1. Find the integral surface of the linear partial differential equation $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ which contains the straight line $x + y = 0, z = 0$
2. Find the integral surface of the linear partial differential equation $2y(z - 3)p + (2x - z)q = y(2x - 3)$ which pass through the circle $z = 0, x^2 + y^2 = 2x$.
3. Find the integral surface of the linear partial differential equation$(x - y)p + (y - x - z)q = z$ through the circle $z = 1, x^2 + y^2 = 1$.
4. Find the integral surface of the linear partial differential equation $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$ which passes through the line $x = 1, y = 0$.
5. Find the integral surface of the linear partial differential equation $4yzp + q + 2y = 0$ and passing throught $y^2 + z^2 = 1, x + z = 2$.

Practical No. - 9: Compatible systems-I

1. Show that $\frac{\partial z}{\partial x} = 7x + 8y - 1$ and $\frac{\partial z}{\partial y} = 9x + 11y - 2$ are not compatible.
2. Show that $\frac{\partial z}{\partial x} = 5x - 7y$ and $\frac{\partial z}{\partial y} = 6x + 8y$ are not compatible.
3. Show that the differential equation $p = x^2 - ay, q = y^2 - ax$ are compatible and find their common solution.
4. Show that $p = 1 + e^{\frac{x}{y}}, q = e^{\frac{x}{y}}(1 - \frac{x}{y})$ are compatible and find their solution.
5. Show that $p = x - \frac{y}{(x^2+y^2)}$, $q = y + \frac{x}{(x^2+y^2)}$ are compatible and find their solution.

Practical No. - 10: Compatible systems-II

1. Show that the equation $xp = yq$ and $z(xp + yq) = 2xy$ are compatible and solve them.
2. Show that the equation $xp - yq = x$ and $x^2p + q = xz$ are compatible and find their solution.
3. Show that the equation $z = px + qy$ is compatible with any equation $f(x, y, z, p, q) = 0$ which is homogeneous in x, y, z.
4. Show that the equation $f(x, y, p, q) = 0, g(x, y, p, q) = 0$are compatible if $\frac{\partial(f,g)}{\partial(x,p)} + \frac{\partial(f,g)}{\partial(y,q)} = 0$.
5. Solve completey the simultaneous equation $z = px + qy$ and $2xy(p^2 + q^2) = z(yp + xq)$.

Practical No. - 11: Charpit's Method-I

1. Find a complete integral of $z = px + qy + p^2 + q^2$.
2. Find the complete integral of $q = 3p^2$.
3. Find the complete integral of $zpq = p + q$..
4. Find the complete integral of $p^2 - y^2q = y^2 - x^2$.
5. Find the complete integral of $z^2(p^2z^2 + q^2) = 1$.

Practical No. - 12: Charpit's Method-II

1. Find the complete integral of.$px + qy = pq$
2. Find the complete integral of.$yzp^2 - q = 0$
3. Find the complete integral of $q = (z + px)z^2$.
4. Find the complete integral of. $f(x, y, z, p, q) = (p^2 + q^2)y - qz$.
5. Find the complete integral of.$f(x, y, z, p, q) = p(1 + q^2) + (b - z)q = 0$

Practical No. - 13: Jacobi's Method-I

1. Find the complete integral of $P_1^3 + p_2^2 + p_3 = 1$.
2. Find the complete integral of $x_3^2p_1^2p_2^2p_3^2 + p_1^2p_2^2 - p_3^2 = 0$.
3. Find the complete integral of $p_1x_1 + p_2x_2 = p_3^2$.

4. Find the complete integral of $2p_1x_1x_3 + 3p_2x_3^2 + p_2^2p_3 = 0$.
5. Find the complete integral of $p_1p_2p_3 = z^3x_1x_2x_3$.

Practical No. - 14: Jacobi's Method-II

1. Find the complete integral of $p_1^2 + p_2p_3 - z(p_2 + p_3) = 0$.
2. Find the complete integral of $2x_1x_3zp_1p_3 + x_2p_2 = 0$.
3. Find the complete integral of.$p_1p_2p_3 + p_4^3x_1x_2x_3x_4^3 = 0$
4. Find the complete integral of.$2x^2y\left(\frac{\partial u}{\partial x}\right)^2\left(\frac{\partial u}{\partial u}\right) = x^2\left(\frac{\partial u}{\partial y}\right) + 2y\left(\frac{\partial u}{\partial x}\right)^2$
5. Solve $p^2x + q^2y = z$ by Jacobi method.

Practical No. - 15: Monge's Method

1. Solve $(r - t)xy - s(x^2 - y^2) = qx - py$.
2. Solve $r + (a + b)s + abt = xy$.
3. Solve $r = a^2t$.
4. Solve $r + ka^2t - 2as = 0$.
5. Solve $(r - s)y + (s - t)x + q - p = 0$.

T.Y. B.Sc. Mathematics (Elective)
Semester-VI
MTH-DSE-362B: Practical on MTH-DSE-361B

Total Hours: 60 **Credits: 2**

Course Objectives	▪ To learn SageMath and its basic applications to mathematics. ▪ To study calculus of one variable with SageMath. ▪ To study integration with SageMath. ▪ To study limits, continuity and partial derivatives with SageMath.
Course Outcomes	After successful completion of this course, students are expected to: ▪ able to solve equations, 2d plot and 3d plot by using SageMath. ▪ able to use SageMath for calculus of one variable. ▪ able to find integration by using SageMath. ▪ able to calculate local maximum and local minimum by using SageMath.

Sr. No.	Contents	Hours
1	Basic in SageMath	4
2	Solving Equations in SageMath	4
3	2D plotting in SageMath	4
4	3D plotting in SageMath	4
5	Calculus with SageMath-I	4
6	Calculus with SageMath-II	4
7	Calculus with SageMath-III	4
8	Applications of Derivatives using SageMath	4
9	Integration with SageMath-I	4
10	Integration with SageMath-II	4
11	Improper Integral with SageMath	4
12	Applications of Integration using SageMath	4
13	Limit and Continuty using SageMath	4
14	Partial Derivatives with SegeMath	4
15	Local Maximum and Minimum with SageMath	4

T.Y. B.Sc. Mathematics (Vocational)
Semester-VI
MTH-VSC-361: Optimization Techniques

Total Hours: 30 **Credits: 2**

<table>
<tr><td>Course Objectives</td><td colspan="2">▪ To study a graphical and simplex method to solve linear programming problem.
▪ To study the initial basic feasible solution and optimum solution of transportation problem.
▪ To study the Hungarian method for solving adssignment problems.
▪ To know the game and various methods for solving it.</td></tr>
<tr><td>Course Outcomes</td><td colspan="2">After successful completion of this course, students are expected to:
▪ solve the linear programming problem by graphical method and simplex method.
▪ apply methods for getting IBFS and optimize it by using MODI method.
▪ find optimum soltion of assignment problems by Hungerian method.
▪ solve the game by using maximin-minimax principal, algebraic method, dominance property and graphical method.</td></tr>
<tr><td>Unit</td><td>Contents</td><td>Hours</td></tr>
<tr><td>Unit I</td><td>Linear Programming Problem (LPP)
▪ Formation of LPP
▪ Solution of LPP by graphical method
▪ Standard and Canonical forms of LPP
▪ Simplex Algorithm
▪ Solution of LPP by simplex method
▪ Artificial variable technique (Big M method)
▪ Special cases in LPP: (a) Unbounded solution, (b) Alternate solution, and (c) Infeasible solution</td><td>7</td></tr>
<tr><td>Unit II</td><td>Transportation Problem (TP)
▪ General Transportation Problem
▪ Transportation Table. Methods for finding IBFS:
  ▪ North –West corner rule
  ▪ Matrix minima method (Least cost method)
  ▪ Vogel's approximation method (VAM)
▪ Optimality test and optimization of solution to TP by U-V method (MODI). Special cases in TP:
  ▪ Alternate solution
  ▪ Maximization TP
  ▪ Unbalanced TP
  ▪ Restricted TP
  ▪ Degeneracy in TP</td><td>8</td></tr>
<tr><td>Unit III</td><td>Assignment Problem (AP)
▪ Mathematical Formulation of Assignment problem
▪ Hungerian method for solving AP
▪ Special cases in AP:
  ▪ Alternate solution
  ▪ Maximization AP
  ▪ Unbalanced AP
  ▪ Restricted AP</td><td>7</td></tr>
</table>

Unit IV	**Game Theory** ▪ Two person-zero sum games ▪ Pure and mixed strategies, value of a game ▪ Maxmin and Minimax principles and saddle point ▪ Solution of 2×2 game by algebraic method and oddment method ▪ Principle of Dominance ▪ Game without saddle points-mixed strategies, Graphical solution of $m \times 2$ and $2 \times n$ games	**8**
Study Resources	▪ KantiSwarup, P. K. Gupta, Man Mohan, Operations Research, S. Chand and Sons, Educational Publishers, New Delhi. Twelfth Edition, 2004. (Chapter No.- 3, 4, 10, 11) ▪ S. D. Sharma and K.Ramnath, Operations Research, Meerut Publication, 2012. ▪ Prem Kumar Gupta, Operations Research, S. Chand and Company pvt Ltd. New Delhi 7th Edition, 2014.	

T.Y. B.Sc. Mathematics (Vocational)
Semester-VI
MTH-VSC-362: Practical on MTH-VSC-361

Total Hours: 60 **Credits: 2**

Course Objectives	▪ To study a graphical and simplex method to solve linear programming problem. ▪ To study the initial basic feasible solution and optimum solution of transportation problem. ▪ To study the Hungarian method for solving adssignment problems. ▪ To know the game and various methods for solving it.	
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Sr. No.	**Contents**	**Hours**
1	Linear Programming Problem-I	**4**
2	Linear Programming Problem-II	**4**
3	Simplex Method-I	**4**
4	Simplex Method-II	**4**
5	Transportation Problem-I	**4**
6	Transportation Problem-II	**4**
7	Transportation Problem-III	**4**
8	Transportation Problem-IV	**4**
9	Assignment Problem-I	**4**
10	Assignment Problem-II	**4**
11	Assignment Problem-III	**4**
12	Assignment Problem-IV	**4**
13	Game Theory-I	**4**
14	Game Theory-II	**4**
15	Game Theory-III	**4**

Practical No. 1: Solution of LPP by graphical method-I

1. Use graphical method to solve the LPP: Min $Z = x_1 + 0.5x_2$
   subject to the constraints $3x_1 + 2x_2 \leq 12$
   $$5x_1 \leq 10$$
   $$x_1 + x_2 \geq 8$$
   $$-x_1 + x_2 \geq 4$$
   $$x_1, x_2 \geq 0$$
2. Use graphical method to solve the LPP: Max $Z = 2x_1 + 4x_2$
   subject to the constraints $x_1 + 2x_2 \leq 5$
   $$x_1 + x_2 \leq 4$$
   $$x_1, x_2 \geq 0$$
   Is this LPP has alternative solution? If yes, find it.
3. Using graphical method, show that the following LPP has unbounded solution.
   Max $Z = 6x_1 + x_2$

subject to the constraints $2x_1 + x_2 \geq 3$
$x_2 - x_1 \geq 0$
$x_1, x_2 \geq 0$

4. Using graphical method show that the following LPP has infeasible solution.
Max $Z = x_1 + x_2$
subject to the constraints $x_1 + x_2 \leq 1$
$-3x_1 + x_2 \geq 3$
$x_1, x_2 \geq 0$
5. Reduce the following LPP to its standard form:
Max $Z = x_1 + x_2 + 4x_3$
subject to the constraints $-2x_1 + 4x_2 \leq 4$
$x_1 + 2x_2 + x_3 \geq 5$
$2x_1 + 3x_2 \leq 2$
$x_1, x_2, x_3 \geq 0$

Practical No. 2: Solution of LPP by graphical method -II

1. Use graphical method to solve the LPP: Max $Z = 2x_1 + 5x_2$
subject to the constraints $x_1 \leq 4$,
$x_2 \leq 3$,
$x_1 + 2x_2 \leq 8$
$x_1, x_2 \geq 0$
2. Use graphical method to solve the LPP: Max $Z = 4x_1 + 3x_2$
subject to the constraints $3x_1 + 4x_2 \leq 24$
$8x_1 + 6x_2 \leq 48$
$x_1 \leq 5$
$x_2 \leq 6$
$x_1, x_2 \geq 0$
Find alternate optimum solution, if it exists.
3. Using graphical method, to solve the LPP: Max $Z = 80x_1 + 120x_2$
subject to the constraints $x_1 + x_2 \leq 9$
$x_1 \geq 2$
$x_2 \geq 3$
$20x_1 + 50x_2 \leq 360$
$x_1, x_2 \geq 0$
4. Using graphical method, to solve the LPP: Min $Z = 20x_1 + 40x_2$
subject to the constraints $36x_1 + 6x_2 \geq 108$
$3x_1 + 12x_2 \geq 36$
$20x_1 + 10x_2 \geq 100$
$x_1, x_2 \geq 0$
5. A firm manufactures two types of products A and B and sale them at a profit of Rs. 2/- on type A and Rs. 3/- on type B. Each product is produced on two machines M_1 and M_2. Type A requires 1 minute of processing time on machine M_1 and 2 minutes of processing time on machine M_2. Type B requires 1 minute of processing time on machine M_1 and 1 minutes of processing time on machine M_2. Machine M_1 is available for not more than 6 hours 40 minutes while machine M_2 is available for 10 hours during any working day. Formulate given problem in LPP and solove it for maximization.

Practical No. 3: Simplex Method-I

1. Use simplex method to solve the LPP: Max $Z = 4x_1 + 10x_2$
subject to the constraints $2x_1 + x_2 \leq 50$
$2x_1 + 5x_2 \leq 100$
$2x_1 + 3x_2 \leq 90$
$x_1 \geq 0, x_2 \geq 0$
2. Using Big-M method show that the following LPP does not possess any feasible solution. Max $Z = 3x_1 + 2x_2$

subject to the constraints $2x_1 + x_2 \leq 2$
$3x_1 + 4x_2 \geq 12$
$x_1 \geq 0, x_2 \geq 0$

3. Using Big-M method show that the following LPP has alternative solution.
Max $Z = 6x_1 + 4x_2$
subject to the constraints $2x_1 + 3x_2 \leq 30$
$3x_1 + 2x_2 \leq 24$
$x_1 + x_2 \geq 3$
$x_1 \geq 0, x_2 \geq 0$
4. Using simplex method solve the LPP: Max $Z = 3x_1 + 4x_2$
subject to the constraints $x_1 + x_2 \leq 4$
$2x_1 + x_2 \leq 5$
$x_1 \geq 0, x_2 \geq 0$
5. Use simplex method to solve the LPP: Max $Z = 3x_1 + 2x_2$
subject to the constraints $x_1 + x_2 \leq 4$
$x_1 - x_2 \leq 2$
$x_1 \geq 0, x_2 \geq 0$

Practical No. 4: Simplex Method-II

1. Use simplex method to solve the LPP: Max $Z = x_1 + x_2$
subject to the constraints $x_1 + 2x_2 \leq 2000$
$x_1 + x_2 \leq 1500$
$x_1 \leq 600$
$x_1, x_2 \geq 0$
2. Use simplex method to solve the LPP: Max $Z = 800x_1 + 600x_2 + 300x_3$
subject to the constraints $10x_1 + 4x_2 + 5x_3 \leq 2000$
$2x_1 + 5x_2 + 4x_3 \leq 1009$
$x_1, x_2, x_3 \geq 0$
3. Solve the LPP by Big-M method: Max $Z = 3x_1 - x_2$
subject to the constraints $2x_1 + x_2 \geq 2$
$x_1 + 3x_2 \leq 3$
$x_2 \leq 4$
$x_1, x_2 \geq 0$
4. Using simplex method solve the LPP: Max $Z = 6x_1 + 9x_2$
subject to the constraints $2x_1 + 2x_2 \leq 24$
$x_1 + 5x_2 \leq 44$
$6x_1 + 2x_2 \leq 60$
$x_1, x_2 \geq 0$
5. Use simplex method to solve the LPP: Max $Z = 3x_1 + 2x_2$
subject to the constraints $-x_1 + 2x_2 \leq 4$
$3x_1 + 2x_2 \leq 14$
$x_1 - x_2 \leq 3$
$x_1, x_2 \geq 0$
Find the alternate solution, if it exists.

Practical No. 5: Transportation Problem (TP)-I

1. Obtain IBFS of TP by using North-West Corner rule

	D	E	F	G	Availability
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Requirements	200	225	275	250	

2. Obtain IBFS of TP by using Matrix Minima Method

	D_1	D_2	D_3	D_4	Capacity
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	0	2	2	1	10
Demand	4	6	8	6	

3. Obtain IBFS of TP by using Vogel's Approximation Method

	D	E	F	G	Availability
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Demand	200	225	275	250	

4. Convert the following unbalanced TP into balanced TP.

		Destinations					
		I	II	III	IV	V	Supply
Sources	A	4	3	26	38	30	160
	B	3	2	34	34	198	280
	C	3	3	24	28	30	240
	Demand	1	1	200	120	240	

5. Obtain IBFS by VAM and solve the transportation problem for minimum cost.

	D_1	D_2	D_3	Supply
S_1	2	7	4	5
S_2	3	3	1	8
S_3	5	4	7	7
S_4	1	6	2	14
Demand	7	9	18	

Practical No. 6: Transportation Problem (TP)-II

1. Find the starting solution to the following TP by using North-West Corner rule

	A	B	C	D	Supply
a	15	51	42	33	23
b	30	42	26	81	44
c	90	40	66	60	33
Demmand	23	31	16	30	

2. Find the starting solution to the following TP by the matrix minima method

	A	B	C	D	Supply
a	15	51	42	33	23
b	30	42	26	81	44
c	90	40	66	60	33
Demmand	23	31	16	30	

3. Find the starting solution to the following TP by the Vogel's approximation method

	A	B	C	D	Supply
a	15	51	42	33	23
b	30	42	26	81	44
c	90	40	66	60	33
Demmand	23	31	16	30	

4. Solve the following TP for minimum cost

A	B	C	Supply

I	50	30	220	1
II	90	45	170	3
III	250	200	50	4
Demand	4	2	2	-

5. Solve the following TP which has initial solution

	A	B	C	D	Supply
a	5	10	4 10	5	10
b	6 20	8	7	2 5	25
c	4 5	2 10	5 5	7	20
Demand	25	10	15	5	

Practical No. 7: Transportation Problem (TP)-III

1. Solve the following transportation problem for maximum profit

	I	II	III	IV	Supply
A	40	25	22	33	100
B	44	35	30	30	30
C	38	38	28	30	70
Demand	40	20	60	30	

2. Obtain an optimal basic feasible solution of the following degeneracy problem

	W_1	W_2	W_3	Available
F_1	7	3	4	2
F_2	2	1	3	3
F_3	3	4	6	5
Demand	4	1	5	

3. Solve the following transportation problem

	A	B	C	D	E	Supply
P	4	1	3	4	4	60
Q	2	3	2	2	3	35
R	3	5	2	4	4	40
Demand	22	45	20	18	30	

4. Obtain the initial basic feasible solution of the following example by north-west corner method as well as by VAM method.

	I	II	III	IV	Supply
A	7	3	5	5	34
B	5	5	7	6	15
C	8	6	6	5	12
D	6	1	6	4	19
Demand	21	25	17	17	

5. Find initial basic feasible solution for following transportation problem by using least cost method

	D_1	D_2	D_3	D_4	Supply
O_1	3	4	6	3	30
O_2	3	5	7	10	50
O_3	2	6	5	7	70
Demand	22	41	44	43	

Practical No. 8: Transportation Problem (TP)-IV

1. Find optimum solution to the following degenerate TP.

	A	B	C	D	Supply
x	10	22	10	20	8
y	15	20	12	8	13
z	20	12	10	15	11
Demand	5	11	8	8	-

2. Solve the following TP for optimal solution.

	A	B	C	D	E	Supply
o_1	8	8	10	6	10	17
o_2	0	8	6	12	14	25
o_3	12	10	8	14	12	24
Demand	10	11	6	16	23	-

3. Solve the following TP which has initial solution

	d_1	d_2	d_3	d_4	Supply
o_1	$\boxed{15}$	$\boxed{51}$	$\boxed{42}$	$\boxed{33}$ 23	23
o_2	$\boxed{30}$ 23	$\boxed{42}$ 5	$\boxed{26}$ 16	$\boxed{81}$	44
o_3	$\boxed{90}$	$\boxed{40}$ 26	$\boxed{66}$	$\boxed{60}$ 7	33
Demand	23	31	16	30	

4. Solve the following transportation problem

	A	B	C	Supply
P	3	2	4	25
Q	8	9	11	50
R	6	5	9	50
S	9	13	13	100
Demand	50	75	50	

5. Find the alternate solution of the following TP, if it exists

	d_1	d_2	d_3	d_4	d_5	Supply
o_1	$\boxed{8}$	$\boxed{8}$ 1	$\boxed{10}$	$\boxed{6}$ 16	$\boxed{10}$	17
o_2	$\boxed{0}$ 10	$\boxed{8}$ 10	$\boxed{6}$ 5	$\boxed{12}$	$\boxed{14}$	25
o_3	$\boxed{12}$	$\boxed{10}$	$\boxed{8}$ 1	$\boxed{14}$	$\boxed{12}$ 23	24
Demand	10	11	6	16	23	

Practical No. 9: Assignment Problem (AP)-I

1. Solve following AP.

	I	II	III	IV
A	2	3	4	5
B	4	5	6	7
C	7	8	9	8
D	3	5	8	4

Is there exist alternative solution? If Yes, Find it.

2. A departmental head has four subordinates and four tasks to be performed. The subordinates differs in efficiency and the tasks differ in their intrinsic difficulty. His estimate, of the time each man would take to perform each task, is given in the matrix below:

	Men			
Tasks	E	F	G	H
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

How should the tasks be allocated, one to a man, so as to minimize total manhours?

3. Solve the following assignment problem for maximum profit.

	1	2	3	4
A	16	10	14	11
B	14	11	15	15
C	15	15	13	12
D	13	12	14	15

4. The following is the cost matrix of assigning 4 clerks to 4 key punching jobs. Find the optimal assignment if clerk I cannot be assigned to job 1:

	Job			
Clerk	I	II	III	IV
a	--	5	2	0
b	4	7	5	6
c	5	8	4	3
d	3	6	6	2

What is the minimum total cost?

5. Convert the following unbalanced AP into balanced AP and solve it for minimization.

	A	B	C
W	9	26	15
X	13	27	6
Y	35	20	15
Z	18	30	20

Practical No. 10: Assignment Problem (AP)-II

1. Use Hungarian method, to solve the following problem for minimum cost.

		Men		
		a	b	c
Tasks	A	8	7	6
	B	5	7	8
	C	6	8	7

2. A marketing manager has 5 salesmen and 5 sales district, considering the capacity of sales man and nature of district. The manager estimates the sales per month (in 100 Rs.) for each salesman in each district would be as follows.

		Sale	District			
		a	b	c	d	e
	x	32	38	40	28	40
Salesman	y	40	24	28	21	36

z	41	27	33	30	37
w	22	38	41	36	36
t	29	33	40	35	39

Find optimum solution to the above AP.

3. Solve the following assignment problem for minimum cost.

	a	b	c	d
A	10	12	19	11
B	5	10	7	8
C	12	14	13	11
D	8	15	11	9

4. Solve the following assignment problem for minimum cost.

	a	b	c	d
A	15	13	14	17
B	11	12	15	13
C	13	12	10	11
D	15	17	14	16

5. Solve the following assignment problem for minimum cost.

	A	B	C	D
I	8	26	17	11
II	13	28	4	26
III	38	19	18	15
IV	19	26	24	16

Practical No. 11: Assignment Problem (AP)-III

1. Solve the following assignment problem for minimum cost

	I	II	III	IV	V
A	1	3	2	3	6
B	2	4	3	1	5
C	5	6	3	4	6
D	3	1	4	2	2
E	1	5	6	5	4

2. A company has four territories and four salesmen available for the assignment. The cells(rupees in lacks) for each salesmen to be assign each territory is given below

		Territories			
		I	II	III	IV
salesmen	A	84	70	56	42
	B	60	50	40	30
	C	60	50	40	30
	D	48	40	32	24

Find the optimum assignment schedule that maximizes total profit.

3. The owner of a small machine shop has four machinists available to assign for jobs for a day. Five jobs are offered with expected profit (in rupees) on each job as follows

		Jobs				
		A	B	C	D	E
machinists	1	62	78	50	101	82
	2	71	84	61	73	59
	3	87	92	111	71	81
	4	48	64	87	77	80

Find the assignment of machinists to the jobs that will result in a maximum profit.

4. Solve the following assignment problem to minimize the total design time

Design time project

Originers		A	B	C	D
	I	12	10	10	8
	II	14	∞	15	11
	III	6	10	16	4
	IV	8	10	9	7

5. There are five pilots and five flights given in the table

Flight No.

Pilot		I	II	III	IV	V
	A	8	2	X	5	4
	B	10	9	2	8	4
	C	5	4	9	6	X
	D	3	6	2	8	7
	E	5	6	10	4	3

Find the optimal solution of the assignment problem.

Practical No. 12: Assignment Problem (AP)-IV

1. Solve the following assignment problem to minimization

	A	B	C	D
I	1	4	6	3
II	9	7	10	9
III	4	5	11	7
IV	8	7	8	5

2. A Company has four machines on which to do three jobs. Each job can be assigne to one and only one machine. The cost of each job on each machine is given in the following table

Machines

Jobs		A	B	C	D
	1	18	24	28	32
	2	8	13	17	19
	3	10	15	19	22

Determine the optimum assignment and minimum cost.

3. Solve the following assignment problem to find maximum total expected sale.

	I	II	III	IV
A	42	35	28	21
B	30	25	20	15
C	30	25	20	15
D	24	20	16	12

4. Solve the following assignment problem

	I	II	III	IV	V
A	4	6	10	5	6
B	7	4	--	5	4
C	--	6	9	6	2
D	9	3	7	2	3

5. Five operators have to be assigned to five machines. Operator A cannot operate machine III and operator C cannot be operate machine IV. The assignment cost are as below.

	I	II	III	IV	V
A	5	5	--	2	6
B	7	4	2	3	4

C	9	3	5	--	3
D	7	2	6	7	2
E	6	5	7	9	1

Find the optimum assignment.

Practical No. 13: Game Theory-I

1. Find the best strategy of each player and the value of game.

Player B

Player A					
	9	3	1	8	0
	6	5	4	6	7
	2	4	3	3	8
	5	6	2	2	1

2. A and B play a game in which each has three coins 5p ,10p and 20p each player selects the point without the knowledge of coin, if the sum of coin is an odd amount, A wins B's coin and if the sum of coin is even then B wins A's coin. Find the best strategy for player A &B and the value of game.
3. Find the ranges of values of P and Q which will render the entry (2, 2)a saddle point for the game

Player B

Player A			
	2	4	5
	10	7	Q
	4	P	6

4. Solve the following 2 X 4 game by graphical method.

Player B

Player A				
	3	3	4	0
	5	4	3	7

5. Solve the following game by graphical method.

Player B

Player A				
	19	6	7	5
	7	3	14	6
	12	8	18	4
	8	7	13	-1

Practical No. 14: Game Theory-II

1. Solve the following game by algebraic method

Player B

Player A		
	5	1
	3	4

2. Solve the following game by graphical method

Player B

Player A				
	2	1	0	-2
	1	0	3	2

3. Solve the following game

Player B

Player A					
	-2	0	0	5	3
	3	2	1	2	2
	-4	-3	0	-2	6
	5	3	-4	2	-6

4. Solve the following game by graphical method.

		Player B	
Player A		1	-3
		3	5
		-1	6
		4	1
		2	2
		-5	0

5. Solve the following game.

$$A\begin{array}{c} B \\ \begin{bmatrix} -2 & 0 & 0 & 5 & 3 \\ 3 & 2 & 1 & 2 & 2 \\ -4 & -3 & 0 & -2 & 6 \\ 5 & 3 & -4 & 2 & -6 \end{bmatrix} \end{array}$$

Practical No. 15: Game Theory-III

1. Solve the following game by arithmetic method

	Player B	
Player A	5	-1
	0	12

2. Solve the following game by using dominance principle

	Player B			
Player A	2	-2	4	1
	6	1	12	3
	-3	2	0	6
	2	-3	7	1

3. Solve the following 5 X 2 game by graphical method.

	Player B	
Player A	2	-4
	-3	5
	-1	6
	4	1
	3	4

4. For the following game

	Player B	
Player A	-4	3
	-3	-7

Find a) saddle point, if exists, b) optimal strategy and c) the value of the game.

5. Using principle of dominance, for the following game

	Player B			
Player A	8	10	9	14
	10	11	8	12
	13	12	14	13

Find a) best strategy for Player A b) best strategy for Player B c) the value of game.

Skills acquired and Job opportunity for the Mathematics students

Skills acquired:

The curriculum is designed to inculcate basic principles of mathematical methods and analysis to apply in various fields of scientific research. The curriculum contains a wide variety of mathematical topics like topology, linear algebra, differential equations, numerical analysis, transformations, operations research, fluid mechanics, functional analysis and mathematical methods. Further the following skills are developed on successful completion:

- critical thinking
- problem solving
- analytical thinking
- quantitative reasoning
- ability to manipulate precise and intricate ideas
- construct logical arguments and expose illogical arguments
- time management
- teamwork
- independence

Job opportunity:

The designed curriculum offers job opportunities like:

- mathematics teacher
- Scientist
- Programmer
- Software professional
- Banker
- Accountant.
- Actuary
- Data analyst
- Engineer
- Investment manager
- Research leading to Ph. D. degree
- Self entrepreneurship