Date :- 01/08/2023

NOTIFICATION

Sub :- CBCS Syllabi of M. Sc. in Mathematics (Sem. I & II)

Ref. :- Decision of the Academic Council at its meeting held on 26/07/2023.

The Syllabi of M. Sc. in Mathematics (First and Second Semesters) as per **NATIONAL EDUCATION POLICY - 2020** and approved by the Academic Council as referred above are hereby notified for implementation with effect from the academic year 2023-24.

Copy of the Syllabi Shall be downloaded from the College Website (www.kcesmjcollege.in)

Sd/-
Chairman,
Board of Studies

To :

1) The Head of the Dept., M. J. College, Jalgaon.
2) The office of the COE, M. J. College, Jalgaon.
3) The office of the Registrar, M. J. College, Jalgaon.

Khandesh College Education Society's

Moolji Jaitha College, Jalgaon

An "Autonomous College"

Affiliated to

Kavayitri Bahinabai Chaudhari
North Maharashtra University, Jalgaon-425001

KNOWLEDGE IS POWER

ESTD. 1945

STRUCTURE AND SYLLABUS

M.Sc. Honours with Research (F.Y.M.Sc. Mathematics)

Under Choice Based Credit System (CBCS)
and
as per NEP-2020 Guidelines

[w.e.f. Academic Year: 2023-24]

Preface

The Moolji Jaitha College (Autonomous) has adopted a department-specific model as per the guidelines of UGC, NEP-2020 and the Government of Maharashtra. The Board of Studies in Mathematics of the college has prepared the syllabus for the first-year postgraduate of Mathematics. The syllabus cultivates theoretical knowledge and applications of different fields of Mathematics. The contents of the syllabus have been prepared to accommodate the fundamental aspects of various disciplines of Mathematics and to build the foundation for various applied sectors of Mathematics. The program will be enlightened the students with the advanced knowledge of Mathematics, which will help to enhance student's employability.

The overall curriculum of two year covers pure mathematics, applied mathematics and computational mathematics with programming. The syllabus is structured to cater the knowledge and skills required in the research field, Industrial Sector and Entrepreneurship etc.. The detailed syllabus of each paper is appended with a list of suggested readings.

Programme Outcomes (PO) for M.Sc. Mathematics Honours with Research

Upon successful completion of this Programme, student will be able to:

PO No.	PO
1	Critical Thinking: Inculcate critical thinking to carry out scientific investigation objectively without being biased with preconceived notions.
2	Knowledge Skill: Equip the student with skills to analyse problems, formulate an hypothesis, evaluate and validate results, and draw reasonable conclusions thereof.
3	Scientific Communication Skills: Imbibe effective scientific and / or technical communication in both oral and writing. Ability to show the importance of the subject as precursor to various scientific developments since the beginning of the civilization.
4	Ethics: Continue to acquire relevant knowledge and skills appropriate to professional activities and demonstrate highest standards of ethical issues in Mathematics.
5	Enlightened Citizenship: Create awareness to become an enlightened citizen with commitment to deliver one's responsibilities within the scope of bestowed rights and privileges.
6	Research Skills: Prepare students for pursuing research or careers in industry in Mathematical sciences. Capability to use appropriate software to solve various problems and to apply programming concepts of C++ , python etc… to various scientific investigations , problem solving and interpretation.

Programme Specific Outcome (PSO) for M.Sc. Mathematics Honours with Research:

After completion of this course, students are expected to:

PSO No.	PSO
1	Demonstrate an understanding of concepts involved in mathematical analysis, algebra and applied mathematics
2	Gain proficiency in mathematical techniques of both pure and applied mathematics and be able to apply the necessary mathematical methods to any scientific problem.
3	Acquire significant knowledge on various aspects related to Linear algebra, Topology, Numerical methods and Differential equations.
4	Learn to work independently as well as a team to formulate appropriate mathematical methods.
5	Develop the ability to understand and practice the morality and ethics regarding scientific Research.
6	Realize the scope of mathematics in enlightening of the society and plan to pursue research which is beneficial to the society.

Credit distribution structure for two years/one-year PG MSc programme

<table>
<tr><th rowspan="2">Level</th><th rowspan="2">Sem</th><th colspan="2">Major (Core) Subjects</th><th rowspan="2">Minor Subjects</th><th rowspan="2">OJT/Int, RP</th><th rowspan="2">Cumulative Credits/Sem</th><th rowspan="2">Degree/ Cumulative Cr.</th></tr>
<tr><th>Mandatory (DSC)</th><th>Elective (DSE)</th></tr>
<tr><td rowspan="3">6.0</td><td>I</td><td>DSC-1 (4T)
DSC-2 (4T)
DSC-3 (4T)
DSC-4 (2P)</td><td>DSE-1(2T) A/B
DSE-2(2P) A/B</td><td>RM (4T)</td><td></td><td>22</td><td rowspan="3">First year PG
OR
One year PG diploma after 3year UG</td></tr>
<tr><td>II</td><td>DSC-5 (4T)
DSC-6 (4T)
DSC-7 (4T)
DSC-8 (2P)</td><td>DSE-3(2T) A/B
DSE-4(2P) A/B</td><td></td><td>OJT/Int (4)</td><td>22</td></tr>
<tr><td>Cum. Cr.</td><td>28</td><td>8</td><td>4</td><td>4</td><td>44</td></tr>
<tr><td></td><td colspan="6">Exit option: PG diploma (44 Credits) after three year UG degree</td><td></td></tr>
<tr><td rowspan="3">6.5</td><td>III</td><td>DSC-9 (4T)
DSC-10 (4T)
DSC-11 (4T)
DSC-12 (2P)</td><td>DSE-5(2T) A/B
DSE-6(2P) A/B</td><td>--</td><td>RP (4)</td><td>22</td><td rowspan="3">Second year PG after 3 year UG
OR
PG degree after 4 year UG</td></tr>
<tr><td>IV</td><td>DSC-13 (4T) DSC-14 (4T)
DSC-15 (2P)
DSC-16 (2P)</td><td>DSE-7(2T) A/B
DSE-8 (2P) A/B</td><td>...</td><td>RP (6)</td><td>22</td></tr>
<tr><td>Cum. Cr.</td><td>54</td><td>16</td><td></td><td>4+10</td><td>88</td></tr>
<tr><td colspan="8">2 Years-4 Sem. PG Degree (80-88 credits) after Three Year UG Degree
or 1 Year-2 Sem PG Degree (40-44 credits) after Four Year UG Degree</td></tr>
</table>

Sem- Semester, **DSC**- Department Specific Course, **DSE**- Department Specific Elective, **T**- Theory, **P**- Practical,
RM- Research Methodology, **OJT**- On Job Training, **Int**- Internship, **RP**- Research Project,
Cum. Cr.:Cumulative Credits

Multiple Entry and Multiple Exit options:

The multiple entry and exit options with the award of UG certificate/ UG diploma/ or three-year degree depending upon the number of credits secured;

<table>
<tr><th rowspan="2">Levels</th><th rowspan="2">Qualification Title</th><th colspan="2">Credit Requirements</th><th rowspan="2">Semester</th><th rowspan="2">Year</th></tr>
<tr><th>Minimum</th><th>Maximum</th></tr>
<tr><td>6.0</td><td>One-year PG Diploma program
after 3 Yr Degree</td><td>40</td><td>44</td><td>2</td><td>1</td></tr>
<tr><td>6.5</td><td>Two-year master’s Degree program
After 3-Yr UG
OrPGDegreeafter 4-Yr UG</td><td>80</td><td>88</td><td>4</td><td>2</td></tr>
</table>

F. Y. M. Sc. Mathematics Course Structure

Semester	Course Module	Credit	Hours/ week	TH/ PR	Code	Title
I	DSC	4	4	TH	MTH-DSC-511	Advanced Metric Spaces
	DSC	4	4	TH	MTH-DSC-512	Algebra
	DSC	4	4	TH	MTH-DSC-513	Differential Equations
	DSE	4	4	TH	MTH-DSE-514(A)	Numerical Methods
	DSE	4	4	TH	MTH-DSE-514(B)	Measure Theory
	DSC	2	4	PR	MTH-DSC-515	Practical course on Algebra and Differential Equations
	RM	4	4	TH	MTH-RM-516	Research Methodology for Mathematics
II	DSC	4	4	TH	MTH-DSC-521	Topology
	DSC	4	4	TH	MTH-DSC-522	Analytic Number Theory
	DSC	4	4	TH	MTH-DSC-523	Complex Analysis
	DSE	4	4	TH	MTH-DSE-524(A)	Linear Algebra
	DSE	4	4	TH	MTH-DSE-524(B)	Calculus of Variation
	DSC	2	4	PR	MTH-DSC-525	Practical course on Analytic Number Theory and Complex Analysis
	OJT /INT	4	8	OJT	MTH-OJT-526	On Job Training/ Internship

DSC	:	Department-Specific Core course
DSE	:	Department-Specific elective
TH	:	Theory
PR	:	Practical

Exam Pattern:

Theory / Practical	Credit	Internal	External
Theory	**4**	**40**	**60**
Theory	**2**	**20**	**30**
Practical	**2**	**20**	**30**

External Theory Examination (30 marks):

- External examination will be of 1.30 hours duration for each theory course. There shall be 3 questions while the tentative pattern of question papers shall be as follows;
- Q1 will be of 6 marks (attempt any 2 out of 3 sub-questions).
- Q2 and Q3 each will be of 12 marks (attempt any 2 out of 3 sub-questions).

External Theory Examination (60 marks):

- External examination will be of 3.00 hours duration for each theory course. There shall be 5 questions while the tentative pattern of question papers shall be as follows;
- Q1 will be of 12 marks (attempt any 3 out of 4 sub-questions).
- Q2, Q3, Q4 and Q5 each will be of 12 marks (attempt any 2 out of 3 sub-questions).

External Practical Examination (30 marks):

- Practical examination shall be conducted by the respective department at the end of the semester. Practical examination will be of minimum 3 hours duration and shall be conducted as per schedule. There shall be 05 marks for journal and *viva-voce*. Certified journal is compulsory to appear for practical examination.

Internal Theory Examination (40 marks):

The Continuous Internal Evaluation for theory papers shall consist of two methods:

1. Continuous & Comprehensive Evaluation (CCE):

CCE will carry a maximum of 30% weightage (30/15 marks) of the total marks for a course. Before the start of the academic session in each semester, the subject teacher should choose any three assessment methods from the following list, with each method carrying 10/5 marks:

i. Individual Assignments
ii. Seminars/Classroom Presentations/Quizzes
iii. Group Discussions/Class Discussion/Group Assignments
iv. Case studies/Case lets
v. Participatory & Industry-Integrated Learning/Field visits

vi. Practical activities/Problem Solving Exercises

vii. Participation in Seminars/Academic Events/Symposia, etc.

viii. Mini Projects/Capstone Projects

ix. Book review/Article review/Article preparation

x. Any other academic activity

xi. Each chosen CCE method shall be based on a particular unit of the syllabus, ensuring that three units of the syllabus are mapped to the CCEs.

2. Internal Assessment Tests (IAT):

IAT will carry a maximum of 10% weightage (10/5 marks) of the total marks for a course. IAT shall be conducted at the end of the semester and will assess the remaining unit of the syllabus that was not covered by the CCEs. The subject teacher is at liberty to decide which units are to be assessed using CCEs and which unit is to be assessed on the basis of IAT.

The overall weightage of Continuous Internal Evaluation (CCE + IAT) shall be 40% of the total marks for the course. The remaining 60% of the marks shall be allocated to the semester-end examinations.

The subject teachers are required to communicate the chosen CCE methods and the corresponding syllabus units to the students at the beginning of the semester to ensure clarity and proper preparation.

Internal Practical Examination (20 marks):

- Internal practical examination of 10 marks will be conducted by department as per schedule given.
- For internal practical examination student must produce the journal of practicals completed along with the completion certificate signed by the concerned teacher and the Head of the department.
- There shall be continuous assessment of 30 marks based on student performance throughout the semester. This assessment can include quizzes, group discussions, presentations and other activities assigned by the faculty during regular practicals. For details refer internal theory examination guidelines.
- Finally 40 (10+30) marks performance of student will be converted into 20 marks.

SEMESTER-I

F.Y.M.Sc.(Mathematics)
SEMESTER-I
MTH-DSC-511: Advanced Metric Spaces

Total Hours: 60 **Credits: 4**

Course objectives	• The basic need of this course is to understand the concepts and applications of metric spaces. • To know Cantor's intersection theorem and Bair's category theorem. • To know the concepts of Isometry, Bolzano-Weierstrass property, Lebsgue number. • To know the Arzela Ascoli Theorem, Contraction principle, concept of connectedness.	
Course outcomes	After successful completion of this course, students are expected to: • understand basic concepts like interior of a set and boundary of a set on Metric spaces. • understand the concepts of sequences and their convergence, Cantor's intersection theorem and Bair's category theorem. • understnd the important theorems in metric spaces and their applications. • understand the Arzela Ascoli Theorem, Contraction principle, concept of connectedness.	
Unit	**Content**	**Hours**
Unit I	Partially ordered sets, well ordered sets, Axiom of choice, Zorn's lemma, Well ordering principle. Revision of Metric spaces, open sets and closed sets, interior of a set, closure of a set and their properties, Boundary of a set.	**15**
Unit II	Sequences in metric spaces, Cauchy's sequences, convergence, Cantor's intersection theorem, No-where dense set, Every where dense set. Completeness, First category, Second category, Bair's category theorem, Completion of metric spaces.	**15**
Unit III	Continuity, Uniform continuity, Homeomorphism, Isometry. Sequentially compact metric spaces, Bolzano-Weierstrass property, Lebsgue number, Compactness, totally bounded sets,	**15**
Unit IV	Separable metric spaces, Arzela Ascoli Theorem. Contraction principle, Existence theorem for differential equations, Connectedness, finite product of connected spaces.	**15**
Study Resources	• Simmon, G. F. *Introduction to Topology and Modern Analysis*. Tata Mc Graw Hill. (Chapter -1: Art.-5, 8; Chapter -2: Art.- 9 to 14; Chapter -4: Art.- 21, 24, 25; Chapter -6: Art.-31; Appendice-1) • Munkers, J. R. (1992). *Topology: A First Course*. Prentice Hall of India Ltd.	

F.Y.M.Sc. (Mathematics)
SEMESTER-I
MTH-DSC-512: Algebra

Total Hours: 60 **Credits: 4**

Course objectives	• To know the concept and applications of finite groups. • To study well known theorems for finite groups: Cauchy's Theorem, Sylow's Theorem, Jordan -Holder Theorem. • To know concepts of particular types of integral domains: ED, PID, UFD. • To know the concept of Northerian rings and Hilbert Basis Theorem.	
Course outcomes	After successful completion of this course, students are expected to: • Understand class equation for finite groups and its applications. • Explain Sylow theory and solvable groups. • Learn Euclidean domains, Principal ideal domains, Unique factorization domains. • Understand the concept of Noetherian rings and the Hilbert Basis Theorem.	
Unit	**Content**	**Hours**
Unit I	Direct products, External direct product of groups, Conjugacy classes, Class equation, Cauchy's Theorem. Sylow $\boldsymbol{p}$-subgroups, Sylow theorems, Solvable group, Normal series.	**15**
Unit II	Composition series, Jordan-Holder Theorem, Greatest common divisor, prime element, irreducible element, Euclidean domain, principal ideal domain, Factorization domain.	**15**
Unit III	Unique Factorization domain, Polynomial rings, Roots of polynomials, Eisenstein's criterion, primitive polynomial.	**15**
Unit IV	Gauss lemma, Gauss theorem, factorization of polynomials, Finitely generated ideals, Chain conditions, Noetherian rings, Hilbert basis theorem.	**15**
Study Resources	• Gopalakrishnan, N. S. (2018). *University Algebra*. Wiley Eastern Limited, New Delhi. (Sec. 1.10, 1.12, 1.13, 1.14, Sec. 2.10, 2.11, 2.12, 2.13, 2.14, 2.15, 2.16). • Gopalakrishnan, N. S. (2016). *Commutative Algebra*. Universities Press (India) Pvt. Ltd. (Sec. 3.1). • Herstein, I. N. (1975). *Topics in Algebra*. John Wiley and Sons, New Delhi. • Jacobson, N. (2012). *Basic Algebra - I*. (2nd ed.). Hindustan Publishing Corporation. • Fraleigh, J. B. (2003). *A first Course in Abstract Algebra*. Pearson. • Bhattacharya, P. B., Jain, S. K., and Nagpaul S. R. (1994). *Basic Abstract Algebra*. Cambridge Press.	

F.Y.M.Sc. (Mathematics)
SEMESTER-I
MTH-DSC-513: Differential Equations

Total Hours: 60 **Credits: 4**

Course objectives	• To understand the concepts and applications of Differential equations. • It will improve problem solving and logical thinking abilities of the students. • To know the concepts of Differential equations to develop mathematical skills. • To know the concepts of Power series solutions and special functions.	
Course outcomes	Upon successful completion of this course the student will be able to: • understand basic concepts on ordinary Differential equations. • understand the concepts of partial differential equations. • know the important theorems and their applications. • know the Legendre polynomials.	
Unit	**Content**	**Hours**
Unit I	**Second Order L.D.E. with constant Coefficients** Basic theory of linear differential equations (L.D.E.),Homogenous L.D.E. with constant coefficients,Method of undetermined coefficients,Variation of Parameters,The Cauchy-Euler equation, theorems on second order homogenous L.D.E	**15**
Unit II	**Power Series Solutions and Special Functions** Introduction to power series, Series solutions of first order equations, Second order Linear equations, ordinary points existence of unique solution at ordinary points, Regular singular points,Frobenius method of series solutions,Indicial equations	**15**
Unit III	**Legendre Polynomials** Series solution,Orthogonality and normalization,Rodruguez formula .Generating function,recursion relations.	**15**
Unit IV	**Partial Differential Equations of the Second Order** Origin, applications in Physics,Linear P.D. E. with constant coefficients,Linear P.D.E. with variable coefficients ,Classification of P.D.E of second order,Characteristics curves,Characteristic curves of second order equations,Characteristics of Equations in three variables,The solution of Linear Hyperbolic Equation ,Riemann – Green's functions	**15**
Study resources	• Shepley, L. R. (1984). *Differential Equations*. John Wiley and sons. • Simmons, G. F. (2017). *Differential Equation*. CRC Press Taylor & Francis Group. • Sneddon, I. (1957). *Elements of Partial Differential Equations*. Mcgraw-Hill Book company Inc.	

F.Y.M.Sc. (Mathematics)
SEMESTER-I
MTH-DSE-514(A): Numerical Methods

Total Hours: 60 **Credits: 4**

Course objectives	• To know direct methods to solve system of linear equations. • To know the concept and applications of numerical methods. • To know the conceprt of numerical solution of ODE. • To know the conceprt of numerical solution of PDE.	
Course outcomes	After successful completion of this course, students are expected to: • understand basic and advanced concept of solving linear equations. • understand the concept of queuing numerical differentiation and integration. • learn important principles and techniques of solving ODE by numerical methods. • learn important principles and techniques of solving PDE by numerical methods.	
Unit	**Content**	**Hours**
Unit I	**System of Linear Equations** Methods of triangularization – Do little algorithm, Crout's method, Inverse of a matrix by Crout's method, Gauss Jordan method for system of linear equations, Iterative methods of Jacobi and Gauss–Seidal, Relaxation method and Convergence.	**15**
Unit II	**Numerical Differentiation and Integration** Numerical differentiation using Forward, Backward, Central differences, Error analysis, higher derivatives of continuous and tabulated functions, Maximum and minimum values of a function, EDifference tables, Richardson's extrapolation, Newton–Cotes Integration formulas, Trapezoidal rule, Simpson's $\frac{1}{3}$-rule, Error Analysis, Romberg integration, Numerical double integration by trapezoidal and Simpson's rules.	**15**
Unit III	**Numerical Solution of ODE (IVP and BVP)** Initial value problems, Numerical Solution of O.D.E using Picard, Taylor series, Modified Euler, Runge-Kutta fourth order methods, Predictor corrector methods, Linear BVP, Shooting method, Alternative method, Finite difference method of linear second order problems, Derivative boundary condition and Solution of tri-diagonal system.	**15**
Unit IV	**Numerical Solution of PDE (BVP)** Introduction, Deriving difference equations, Numerical solution of elliptic equations, Leibnitz's iteration method for Laplace equation and Poisson's equation, Solution of Heat equation, Bendor-Schmidt method, Crank-Nicholson method, Hyperbolic equations, Finite difference method and starting values.	**15**
Study Resources	• Jain, M. K., Iyengar, S. R. K., & Jain, R. K. (2014). *Numerical methods for Scientific and Engineering Computation*. New Age international Publishers. • Vedamurthy, V. N., & Iyengar N. Ch. S. N. (1998). *Numerical methods*. Vikash Publishing House. • Balagurswamy, E. (2017). *Numerical Methods*. Tata McGraw-Hill. • Sastry, S. S. (2012). *Introductory methods of Numerical Analysis*. Prentice Hall India. • Gerald, C., & Wheatley, O. (2003). *Applied Numerical Analysis* (7th ed.). Addison Publishing company.	

F.Y.M.Sc. (Mathematics)
SEMESTER-I
MTH-DSE-514(B): Measure Theory

Total Hours: 60 **Credits: 4**

Course objectives	• To know the fundamental knowledge of measure theory. • To study Countable sets, Zorn's lemma, Well Ordering principle, Vitali covering theorem (lemma), Fundamental theorem for integral calculus for Lebesgue integral. • To know Measurable sets and Measurable functions and their applications. • To study the concept of function of boundedvariation.	
Course outcomes	After successful completion of this course, students are expected to: • understand Lebesgue outer measure, Riemann and Lebesgue integrals. • explain the Schroeder- Bernstein theorem, Cantors theorem and the continuum Hypothesis, Lebesgue differentiation theorem. • learn the concept of Functions of bounded variation, differentiation of monotone function. • learn the concept of fundamental theorem of integral calculus for Lebesgue integral.	
Unit	**Content**	**Hours**
Unit I	**Countable and Uncountable Sets** Countable and uncountable sets, Infinite sets and the axioms of choice, Cardinal numbers and their arithmetic, Schroeder- Bernstein theorem, Cantors theorem and the continuum Hypothesis, Zorn's lemma, Well Ordering principle, Cantor set, Cantor like sets, The Lebesgue functions.	**15**
Unit II	**Measure on the Real Line** Lebesgue outer measure, Measurable sets, Regularity, Measurable functions, Borel sets and Lebesgue measurability.	**15**
Unit III	**Integration of Functions of a Real Variable** Integration of nonnegative function, The general integral, Integration of series, Riemann and Lebesgue integrals.	**15**
Unit IV	**Differentiation** The four derivatives, Functions of bounded variation, Lebesgue differentiation theorem, Differentiation and Integration, Vitali covering theorem (lemma), Fundamental theorem for integral calculus for Lebesgue integral, Absolutely continuous functions.	**15**
Study Resources	• Barra, G. De. (2000). *Measure Theory and Integration*. New Age International Private Limited. • Royden, H. L. (2009). *Real analysis* (4th ed.). Prentice Hall of India Private Limited. • Halmos, P. R. (1914). *Measure Theory*. (2nd ed.). Springer International, Narosa Publishing House.	

F.Y.M.Sc. (Mathematics)
SEMESTER-I
MTH-DSC-515: Practical Course on Algebra and Differential Equations

Total Hours: 30 **Credits: 2**

Course objectives	- To know the concept and applications of Finite groups, Cauchy's Theorem, Sylow's Theorem, Jordan -Holder Theorem.. - To know concepts of particular types of integral domains: ED, PID, UFD, Noetherian rings. - To know the concepts of ordinary differential equations and partial differential equations. - To know the concepts of Power series solutions and Legendre polynomials	
Course outcomes	Upon successful completion of this course the student will be able to: - Understand class equation, Finite groups, Cauchy's Theorem, Sylow's Theorem, Jordan -Holder Theorem and their applications. - Learn the applications Euclidean domains, Principal ideal domains, Unique factorization domains, Noetherian rings and the Hilbert Basis Theorem. - understand basic concepts on ordinary differential equations and partial differential equations. - understand the power series solutions and Legendre polynomials.	
Practical No.	**Content**	**Hours**
1	**Practical No.1: Finite Groups** 1. Find all the distinct conjugate classes in the group $(\mathbb{Z}_6, +_6)$. 2. Find the centre of the group of $\boldsymbol{S_n}$. 3. Verify the class equation for $\boldsymbol{S_3}$. 4. Find all sylow 2 and sylow 3 subgroups of $\boldsymbol{S_3}$. 5. Let $\boldsymbol{G}$ be a group. Suppose that $\boldsymbol{G}$ acts on $\boldsymbol{S}$ by conjugation. Show that $\boldsymbol{G}$-orbit of an element $\boldsymbol{a} \in \boldsymbol{G}$ is same as conjugate class of $\boldsymbol{a}$ in $\boldsymbol{G}$. 6. Let $\boldsymbol{G}$ be a group which acts on $\boldsymbol{G}$ by conjugation. Show that $\boldsymbol{Stab(a)} = \boldsymbol{N(a)}$, for all $\boldsymbol{a} \in \boldsymbol{G}$. 7. Show that a group of order 345 has a subgroup of index 5. 8. Prove or disprove : A group of order $\boldsymbol{p^2}$ is always abelian where $\boldsymbol{p}$ is prime. 9. Prove or disprove : A group of order $\boldsymbol{p^3}$ is always abelian where $\boldsymbol{p}$ is prime. 10. If $\boldsymbol{G}$ is a finite group such that every element of $\boldsymbol{G}$ is of order of some power of 7, then show that $\boldsymbol{o(G) = 7^n}$ for some $\boldsymbol{n \geq 1}$.	**4**
2	**Practical No.2: Sylow-Theorems, Solvable Groups and Composition Series** 1. Prove that a group of order 35 is cyclic. 2. Show that a group of order 56 cannot be simple. 3. Show that a group of order 143 cannot be simple. 4. True or false? Justify : $\boldsymbol{S_4}$ is solvable group. 5. Show that every $\boldsymbol{p}$-group is solvable.	**4**

	6. Show that $\boldsymbol{S_3}$ is solvable but not abelian. 7. Show that $\boldsymbol{S_n}$ is not solvable group for $\boldsymbol{n \geq 5}$. 8. Show that $\mathbb{Z}_6 \supseteq \langle \bar{3} \rangle \supseteq \{\bar{0}\}$ is a composition series for group $(\mathbb{Z}_6, +_6)$. 9. True or false? Justify : Every group has a composition series. 10. Write down two composition series for the group $\boldsymbol{G} = \mathbb{Z}_{30,}$ and verify that they are equivalent.	
3	**Practical No.3: Integral Domains, Euclidean Domains and PID** 1. Show that $\mathbf{1 + 2\sqrt{-5}}$ is an irreducible in a ring $\boldsymbol{R} = \{\boldsymbol{a} + \boldsymbol{b}\sqrt{-5}\ \boldsymbol{a}, \boldsymbol{b} \in \mathbb{Z}\}$. 2. Give an example of an irreducible element which is not prime. 3. Find all units in $\mathbb{Z}(\sqrt{-5})$. 4. Show that associate of an irreducible element in an integral domain is an irreducible element. 5. Give an example of an integral domain which is not a field. 6. Prove that the ring of Gaussian integers is an Euclidean domain. 7. Find $\boldsymbol{g.c.d.}(\mathbf{1 + 2\sqrt{-5}}, \mathbf{3})$ in integral domain $\mathbb{Z}(\sqrt{-5})$. 8. Show that the $\boldsymbol{g.c.d.}$ of two elements may not exist. 9. Show that product of two PID need not be PID. 10. Show that by an example that quotient ring of a PID is not PID.	**4**
4	**Practical No.4: UFD, Polynomial Rings and Noetherian Rings** 1. Show that every irreducible element of UFD $\boldsymbol{R}$ is prime element. 2. Give an example of ring $\boldsymbol{R}$ and polynomial $\boldsymbol{f(x)} \in \boldsymbol{R}[\boldsymbol{x}]$ of degree $\boldsymbol{n}$ such that $\boldsymbol{f(x)}$ has more than $\boldsymbol{n}$ roots in $\boldsymbol{R}$. 3. Show that $\boldsymbol{f(x) = 1 + x + x^2 + x^3 + x^4} \in \mathbb{Z}[\boldsymbol{x}]$ is irreducible over $\mathbb{Q}$. 4. Show that $\sqrt[4]{\mathbf{7}}$ is an irrational number. 5. Give an example of $\boldsymbol{UFD}$ which is not a $\boldsymbol{PID}$. 6. True or False? Justify. $\mathbb{Z}_{17}[\boldsymbol{x}]$ is a Noetherian ring. 7. Prove or disprove: Subring of a Noetherian ring is Noetherian. 8. True or False? Justify : Every Noetherian ring is an Eucidean domain. 9. Prove or disprove : Homomorphic image of a Noetherian ring is Noetherian. 10. Give an example of Noetherian ring which is not a $\boldsymbol{PID}$.	**3**
5	**Practical No. 5: Second Order Differential Equation with Constant and Undetermined Coefficients** 1. Solve $(\boldsymbol{D^2 + a^2})\boldsymbol{y} = \boldsymbol{cotax}$. 2. Solve $(\boldsymbol{D^2 - 4D + 4})\boldsymbol{y} = \boldsymbol{8x^2e^{2x}sin2x}$. 3. Solve $\boldsymbol{\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = -37sin3x}$ and find the value of y when $\boldsymbol{x} =$	**4**

	$\frac{\pi}{2}$ if it is given that $y = 3$ and $\frac{dy}{dx} = 0$ when $x = 0$. 4. Solve $(D^2 + 4D + 4)y = 2cosh2x$. 5. Solve $(D^2 - 4D + 4)y = x^3e^{2x} + sinhx$. 6. Solve $(D^2 + 4)y = x^2$ by method undermine coefficients. 7. Solve $y_2 + 2y_1 + y = x - e^x$ by the method of undermine coefficients. 8. Solve $y_2 - 2y_1 + y = x^2$ by the method of undermine coifficients. 9. Solve $y_2 - 2y_1 + 3y = cosx + x^2$ by the method of undermine coifficients. 10. Solve $(D^2 + 1)y = 12cos2x$ by method undermine coefficients.	
6	**Practical No. 6: Variation of Parameter and Euler's Equation** 1. Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x sinx$ by using variation of parameter. 2. Solve $y_2 + y = secx$ by using variation of parameter. 3. Solve $(D^2 - 4D + 4)y = x^3e^x + sinhx$ by using variation of parameter. 4. Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x sinx$ with $y(0) = 0$ and $(\frac{dy}{dx})_{x=0} = 0$ by using variation of parameter. 5. Solve $y_2 - y = \frac{2}{1+e^x}$ by using variation of parameter. 6. Solve $(D^2 - 2D + 2)y = e^x tanx$ by using variation of parameter. 7. Solve the Couchy-Euler's equation $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 0$. 8. Solve the Couchy-Euler's equation $(x^2D^2 - 3xD + 4)y = 2x^2$. 9. Solve the Couchy-Euler's equation $x^2\frac{d^2y}{dx^2} + 5x\frac{dx}{dy} + 4y = xlogx$. 10. Solve the Couchy-Euler's equation $x^2\frac{d^2y}{dx^2} - 2x\frac{dx}{dy} + 2y = logx$.	**4**
7	**Practical No. 7: Solution of Power Series and Special Functions** 1. Using power series solve $(1 + x^2)y'' + 2xy' - 2y = 0$. 2. Using power series solve $y' + y = 0$. 3. Using power series solve $y' = 2xy$. 4. Solve $y' = y$ by using power series method. 5. Solve $x^3y'' + x^2y' + xy = 0$ by using power series. 6. Solve $x^2y'' - 3xy' + (4x + 4)y = 0$ by using Frobenius method. 7. Find the indical equation and its roots for the differential equation $xy'' + 2y' + xy = 0$. 8. Solve $2x^2y'' + x(2x + 1)y' - y = 0$ by using Frobenius method. 9. Find the indical equation and its roots for the differential equation $4xy'' + 2y' + xy = 0$	**4**

	10. Find the indical equation and its roots for the differential equation $8xy'' + 4y' + xy = 0$.	
8	**Practical No. 8: Singular Points and Legendre's Polynomial** 1. Show that $x = 0$ is regular singular point $x^2y'' + xy' + (x^2 - n^2)y = 0$ where n is constant. 2. Show that $x = 0$ is ordinary point and $x = \pm 1$ are regular singular point 3. $(1 - x^2)y'' - 2xy' + l(l + 1)y = 0$ where l is constant. 4. Show that regular singular point of $x^2(x - 2)y'' + 2(x - 2)y' + (x + 3)y$. 5. Find general solution of $(1 + x^2)y'' + 2xy' - 2y = 0$ in term of power series. 6. Find the characteristic curves of $4u_{xx} + 5u_{xy} + u_x + u_y = 0$. 7. Show that legendre's polynomial are orthogonal. 8. Prove that $\int_{-1}^{1} p_n(x)dx = 0$ if $n \geq 0$. 9. Show that $(1 - x^2)p'_n = (n + 1)(xp_n - p_{n+1})$. 10. Show that $(n + 1)p_n = p'_{n+1} - xp'_n$.	**3**
Study Resources	• Gopalakrishnan, N. S. (2018). *University Algebra*. Wiley Eastern Limited, New Delhi. (Sec. 1.10, 1.12, 1.13, 1.14, Sec. 2.10, 2.11, 2.12, 2.13, 2.14, 2.15, 2.16). • Gopalakrishnan, N. S. (2016). *Commutative Algebra*. Universities Press (India) Pvt. Ltd. (Sec. 3.1). • Shepley, L. R. (1984). *Differential Equations by John Wiley and sons*, New York. • Simmons, G. F. (2017). *Differential Equations*. CRC Press Taylor & Francis Group, Boca Raton London, New York. • Sneddon, I. (1957). *Elements of Partial Differential Equations*. Mcgraw-Hill Book company Inc, New York.	

F.Y.M.Sc. (Mathematics)
SEMESTER-I
MTH-RM-516: Research Methodology for Mathematics

Total Hours: 60 **Credits: 4**

Course objectives	To make the students familiar, • To learn the basics of science, scientific research its importance. • To learn the Ethics and plagiarism precautions to be taken while doing research. • To the detailed referencing and literature review procedure before beginning the research. • To understand the process of writing research papers, research project report and research proposal. • To know the source of information and tools for digital literature survey.	
Course outcomes	On completion of this course, the student will be able to: • Students will understand the basic concept of science and scientific research. • Learn and follow the ethical guidelines while doing research avoid plagiarism in research publications. • Write a comprehensive literature review on a given research topic. • Write a crisp research proposal or research project independently.	
Unit	**Content**	**Hours**
Unit I	What is Science? Characteristics of Science, Technology and techno-science, Meaning of Research, Characteristics and types of research, Importance of research activities. Principles of quality research work, Problems in research, Scientific attitude and temper, Qualities of good researcher, Scientific community, Non-science and Pseudoscience, Scientific realism.	**15**
Unit II	Introduction, Research planning and design, Selection of research topic, Criteria for good research problem, Source of research Idea, Principles of good research, Criteria of good research, Guidelines for research skill and awareness, Research validity and reliability, Artefact and bias in research. Scientific methodology: Rules and principles of scientific methods, Research methods versus methodology, Hypothesis and testing of hypothesis. Research ethics: Principles and values. Plagiarism: its types and how to avoid it.	**15**
Unit III	**Literature Survey** Literature review, Approaching the literature, Scholarly literature, Data provenance and evaluation, Intellectual property. **Sources of Information**: Primary, Secondary, Tertiary sources, Patents, Journals (Print and e-journal), Type of Journals, Conference Proceedings. Journal Impact Factor, Citation index, h-index. **Understanding of Literature:** Reading A Scientific Paper, Abstracts, Current titles, Reviews, Monographs, Books, Current contents, Cross referencing, Indian patent database. **Tools for Digital Literature Survey:** Scientific databases, e-journals, INFLIBNET, Shodsindhu, Shodhganga, Google/Google Scholar, ResearchGate, PubMed, finding and citing Information. **Scientific Writing:** Introduction to scientific writing, writing science laboratory Notebook.	**15**
Unit IV	**Writing Research Paper:** Title, Abstracts, Keywords, Introduction, Material and Methods, Results and discussion, Conclusion, Acknowledgement, Study Resources and Supplementary data Difference between research communication and Review article, Reply to Referee comments for science research paper. Preparation of Poster and Oral Presentation	**15**

	Writing Proposals: Research grant and its various components. **Advanced Scientific Tools and Laboratory Safety:** A) **Advanced Tools**: Tools for citing and referencing: Mendeley, Zotero, Endnote etc. **Styles of Referencing:** Referencing from reputed publishing houses National and International. Online searching Databases: SciFinder, Scopus, Web of Science, ACM Digital Library, ProQuest Biological Sciences (All the databases only introduction). B**) Laboratory Safety:** Laboratory safety, Laboratory manual, Lab as a safe place: habits, Cause of accidents and What to do in case of an accident, Personal protective equipment, Emergency equipment for general purpose. Laboratory ventilation.	
Study Resources	• Prathapan, K. (2019). *Research Methodology for Scientific Research*. Wiley International Pvt. Ltd., New Delhi – 110002. (Unit 1: pages-1-24 , 49-54, 148-180, Unit-2: pages-1-24, 55-92, 180-229 and 233-262) • Pruzan, P. (2016). *Research Methodology: The Aims, Practices and Ethics of Science*. Springer International Publishing. (Unit 1: pages-1-71) • Kothari, C. R. (2004). *Research Methodology: Methods and Techniques* (3rd ed.). Published by New Age International (P) Ltd., Publishers. (Unit 1: pages-1-21, 24-52) • Pecorari, D. (2013). Teaching to Avoid Plagiarism How To Promote Good Source. Use-Open University Press. (Unit-1: pages-299-317) • Smith, M. B., and March, J. (2013). APPENDIX A: The Literature of Organic Chemistry March's Advanced Organic Chemistry: Reactions, Mechanisms, and Structure. (7th ed.). John Wiley & Sons, Inc. (Unit-1: pages-1569-1603) • JoaquínIsac-García, José A. Dobado, Francisco G. Calvo-Flores, HenarMartínez-García (2016). Experimental Organic Chemistry laboratory manual, Academic Press. (Unit-2: pages-29-43) • Tyowua, A. T. (2023). A Practical Guide to Scientific Writing in Chemistry Scientific Papers, Research Grants and Book Proposals. CRC Press is an imprint of Taylor & Francis Group, LLC. (Unit 2: Relavent pages) • Currano, J. N., Roth, D. L. (2014). Chemical Information for Chemists: A Primer. The Royal Society of Chemistry. (Unit 3: Relavent pages) • Handbook of Safety in Science Laboratories Education Bureau Kowloon Tong Education Services Centre, Hong Kong (2013). (Unit 3: Relavent pages) • Alvi, M. H. (2016). A Manual for Referencing Styles in Research. (Unit 3: Relavent pages) • https://academic.oup.com/pages/authoring/books/preparing-your-manuscript/referencing-styles • https://revvitysignals.com/products/research/chemdraw • LaTeX Beginner's Guide, Stefan Kottwitz, Packt Publishing, http://static.latexstudio.net/wp-content/uploads/2015/03/LaTeX_Beginners_Guide.pdf • Falagas, M. E., Pitsouni, E. I., Malietzis, G. A. and Pappas, G. (2008), Comparison of PubMed, Scopus, Web of Science, and Google Scholar:	

strengths and weaknesses. The FASEB Journal, 22: 338-342. https://doi.org/10.1096/fj.07-9492LSF

- Alvi, M. H. Plagiarism, Citation and Referencing: Issues and Styles, A Manual for Referencing Styles in Research. DOI: 10.13140/RG.2.1.5149.6408 http://bit.ly/46nFwYi
- Gyankosh, D. K. (2013). Citation tools: Easing up the researchers' efforts. The Journal of Lib. & Info. Management, 4(2), Jul-Dec, 2013.
- Citation Management: How to use citation managers such as EndNote and Zotero.
- URL: https://guides.lib.uchicago.edu/citationmanagement
- https://pubs.acs.org/doi/full/10.1021/acsguide.40303
- https://edu.rsc.org/resources/how-to-reference-using-the-rsc-style/1664.article
- https://www.springer.com/gp/authors-editors/journal-author/journal-author-helpdesk/preparation/1276
- https://service.elsevier.com/app/answers/detail/a_id/28224/supporthub/publishing/
- EndNote: A comprehensive guide to the reference management software EndNote. URL: https://aut.ac.nz.libguides.com/endnote
- Zotero: Learn how to use the reference management software Zotero. URL: https://aut.ac.nz.libguides.com/zotero
- Mendeley: Learn how to use the reference management programmeMendeley. URL: https://aut.ac.nz.libguides.com/mendeley
- Grammarly User Guide,
- https://bpb-apse2.wpmucdn.com/blogs.auckland.ac.nz/dist/3/316/files/2020/02/Grammarly-Manual-Feb-2020-1.pdf
- Online Resources: Publishers, Chemical Societies, Electronic Journals etc.: https://www-jmg.ch.cam.ac.uk/data/c2k/cj/
- https://scholar.google.com/
- https://shodhganga.inflibnet.ac.in/
- https://patents.google.com/
- https://ipindia.gov.in/history-of-indian-patent-system.htm
- https://www.cas.org/about-us
- https://clarivate.com/products/scientific-and-academic-research/research-discovery-and-workflow-solutions/webofscience-platform/
- https://www.mendeley.com/guides

SEMESTER-II

F.Y.M.Sc. (Mathematics)
SEMESTER-II
MTH-DSC-521: Topology

Total Hours: 60 **Credits: 4**

Course objectives	• Students will learn the concept of topology and topology generated by basis. • Students will learn, subspaces, closed sets and limit points of a set. • Students will learn continuous functions on topological spaces, product topology, metric topology and quotient topology. • Students will learn connectedness of a set, compactness and separation axioms.	
Course outcomes	After successful completion of this course, students are expected to: • understand the definition of topology, examples, basis and order topology. • understand subspaces, closed sets and limit points of a set. • understand continuous functions on topological spaces, product topology, metric topology and quotient topology. • understand connectedness of a set, compact ness and separation axioms.	
Unit	**Content**	**Hours**
Unit I	Topological spaces, Basis for a topology, Order topology, Product topology on $\boldsymbol{X} \times \boldsymbol{Y}$, Subspace topology, Closed sets, Limit Points and Continuous functions.	**15**
Unit II	Metric topology, Quotient topology, Connected spaces, Connected subspaces of the real line, Components and Local connectedness.	**15**
Unit III	Compact spaces, Compact subspaces of the real line, Limit point compactness and Local compactness.	**15**
Unit IV	Countability axioms, Separation axioms, Normal spaces, Urysohn lemma and Tietze extension theorem.	**15**
Study Resources	• Munkres, J. R. (2000). *Topology* (2nd ed.). Prentice Hall. (Sections 12- 17, 18- 20, 22- 33). • Joshi, K. D. (2017). *Introduction to general topology* (2nd ed.). New Age International Private Limited. • Patty, C. W. (2008). *Foundations of Topology* (2nd ed.). Jones and Bartlett Publishers.	

F.Y.M.Sc. (Mathematics)
SEMESTER-II
MTH-DSC-522: Analytic Number Theory

Total Hours: 60 **Credits: 4**

Course objectives	• To know concept of arithmetic functions. • To study congruences and quadratic residues. • To know the concepts of primitive root theory. • To know the concepts of Lagrange's theorem and Diophantine equations.	
Course outcomes	After successful completion of this course, students are expected to: • Understand the concept of Mobius function μ(n), The Euler totientfunctionφ(n), Mangolt function Λ(n), Liouvilles function λ(n), Thedivisor function σ(n), Bell series. • Explain Residue classes, Lagrange's theorem and its applications, Polynomial congruences with prime power moduli. • Learn Quadratic residues, existence and non- existence of primitive roots. • Understand the applications of Lagrange's theorem and Diophantine equations.	
Unit	**Content**	**Hours**
Unit I	The Mobius function μ(n), The Euler totient function φ(n), Dirichlet product of arithmetic functions, Dirichlet inverses and the Mobius inversion formula. The Mangolt function Λ(n), Multiplicative functions.	**15**
Unit II	Dirichlet multiplication, The inverse of a completely, Liouvilles function$\boldsymbol{\lambda(n)}$, The divisor function $\boldsymbol{\sigma(n)}$, Generalized convolutions. Formal power series, Bell series of an arithmetical function, Bell series and Dirichlet multiplication, Derivatives of arithmetical functions, The Selbergidentity.	**15**
Unit III	Residue classes, Complete and reduced residue systems and Euler- Fermat's theorem, Polynomial congruences $mod\ p$. Lagrange's theorem and its applications, Polynomial congruences with prime power moduli. The principle of cross classification, Quadratic residues, Legendre's symbol and its properties, Evaluation of $(-1\|p)$ and $(2\|p)$	**15**
Unit IV	Gauss lemma, The Quadratic Reciprocity law and its applications, The Jacobi Symbol. Applications to Diophantine equations, The exponent of a number modulo m, Primitive roots,Primitive roots and reduced residue systems, The non- existence of primitive roots $mod\ p^n$ and $2p^n$ for odd primes p and $n \geq 1$. The nonexistenceof primitive roots in the remaining cases. The number of primitive roots $mod\ m$. The primitive roots and quadratic residues. The index calculus.	**15**
Study Resources	• Apostol, T. M. (1972). *Introduction to Analytic Number Theory* (Student ed.). Springer International. (Sec. 2.1 - 2.19, Sec. 5.2, 5.4, 5.5, 5.6, 5.9, 5.10, Sec. 9.1 to 9.8, Sec. 10.1to 10.10). • Burton, D. M. (1980). *Elementary Number Theory*. Universal Book Stall. • Silverman, J. H. (2001). *A Friendly Introduction to Number Theory* (2nd ed.). Prentice Hall. • Niven, I., Zuckerman, H. S., and Montgomery, H. L. (1991). *An introduction to the theory of numbers* (5th ed.). John Wiley and sons.	

F.Y.M.Sc. (Mathematics)
SEMESTER-II
MTH-DSC-523: Complex Analysis

Total Hours: 60 **Credits: 4**

Course objectives	• To make student aware of advances in complex analysis. • To know Mobius transformation and conformal mappings. • To Improve the logical thinking ability to find applications. • To gain the knowledge of singularities.	
Course outcomes	After successful completion of this course, students are expected to: • acquire useful knowledge of complex analysis • understand the concept of power series about complex analysis • solve the complex integration in various forms • prepare themselves for competitive examinations: SET, NET, GATE etc.	
Unit	**Content**	**Hours**
Unit I	Power series, Analytic functions, Branch of a logarithm, Mobius (Bilinear) Transformations and Conformal Mappings, Riemann- Stieltjes Integrals, Power Series representation of analytic functions, Taylor's Theorem, Cauchy's Estimate.	**15**
Unit II	Zeros of an analytic function, Liouville's theorem, Fundamental theorem of algebra, Maximum Modulus Theorem, Index of a closed curve, Cauchy's theorem, Cauchy's Integral Formula, Higher Order derivatives, Morera's Theorem, Homotopic version of Cauchy's Theorem and simple connectivity.	**15**
Unit III	Counting of Zeros, The Open mapping theorem, Goursat's theorem, Singularities, Classification of Singularities, Laurent's series, Casorati-Weierstrass theorem, Residues, Cauchy's residue theorem, Evaluation ofintegrals, Meromorphic functions.	**15**
Unit IV	Argument principle, Rouche's theorem, Schwartz lemma, Convex functions and Hadamard's three circles theorem, The space of continuous functions, Spaces of analytic functions, The Riemann mapping theorem.	**15**
Study Resources	• Conway, J. B. (1995). *Functions of One Complex variable* (2nd ed.). Springer Int. • Ponnusammy, S., & Silverman, H. (2006). *Complex Variables with Applications*. Birkhauser. • Ponnusammy, S.(2022). *Foundations of Complex Analysis* (2nd ed.). Narosa Publishing House. • Ahlfors, L. V. (1996). *Complex Analysis*. McGraw-Hill Book Co..	

F.Y.M.Sc. (Mathematics)
SEMESTER-II
MTH-DSE-524(A): Linear Algebra

Total Hours: 60 **Credits: 4**

Course objectives	• To develop skills and to acquire knowledge of Linear Algebra, Rings and Modules. • To prepare students for further courses in mathematics and/or related disciplines (e.g. Commutative algebra, homological algebra, etc.). • To develop the ability to demonstrate underlying principles of the subject and the ability to solve unseen mathematical problems. • To know the concept of Jordan and Rational canonical forms.	
Course outcomes	After successful completion of this course, students are expected to: • Understand and interpret the concepts of modules and submodules, homomorphism and isomorphism in modules, types of modules. • Understand structure theorems and group theorem. • Understand the concepts of Jordan and Rational canonical forms and use them to solve problems involved in matrix theory and computer algebra. • Understand the concepts of Local rings and modules, Noetherian modules, Primary decomposition for modules.	
Unit	**Content**	**Hours**
Unit I	Modules, Submodules, R- homomorphism, Isomorphism.	**15**
Unit II	Cyclic modules, Faithful modules, Direct sum of modules, free modules, Rank.	**15**
Unit III	Torsion and Torsion free modules, Structure theorem for finitely generated modules over PID, Application to group Theorem.	**15**
Unit IV	Jordan and Rational Canonical forms, Noetherian Modules, Primary decomposition for modules.	**15**
Study Resources	• Gopalkrishnan, N. S.(1988). *University Algebra*. Wiley–Eastern. (Sec. 3.6, 3.7, Sec. 5.10). • Musli, C. S. (2001). *Introduction to Rings & Modules*. Cambridge University Press. (Sec.2.1, 2.2, 2.3, 3.2) • Herstein, I. N. (1988). *Topics in Algebra*. Wiley–Eastern. • Atiyah, M. F., and MacDonald, I. G. (2018). *Algebra*. CRC Press, Boca Raton. • Lambek, J. (1966). *Lectures on Rings and Modules*. Blaisdell Publications, Massachusetts.	

F.Y.M.Sc. (Mathematics)
SEMESTER-II
MTH-DSE-524(B): Calculus of Variation

Total Hours: 60 **Credits: 4**

Course objectives	• To know the concepts of Euler's equations and functionals. • To know the Geodesic and Rayleigh-Ritz method. • To know the transversality and generalized boundary. • To know Jacobi condition, Legendre condition and applications.	
Course outcomes	After successful completion of this course, students are expected to: • understand basic concepts on functionals. • understand the concepts of variational principles. • know the important theorems and their applications. • solve various types of problems of extrema and constrained motion.	
Unit	**Content**	**Hours**
Unit I	Functionals, Euler's Equations, Another form and cases of Euler's Equation, Necessary and sufficient conditions for extremums, Functionals dependent on Higher order derivatives.	**15**
Unit II	Extension of the variational cases, Isoperimetric Problems, Geodesic, Rayleigh-Ritz Method, Lagrange's Equation, Invariance of Euler Equation.	**15**
Unit III	Transversality Condition, Variational Problem with Moving Boundary in Implicit form, Basic Problem with variable end, Generalized Boundary and Transversality condition for the variable end points, One sided variation, Extremals with corners.	**15**
Unit IV	Jacobi Condition, Wierstrass Function, Sufficient condition for extremum, Legendre condition, Application of Calculus of Variation.	**15**
Study Resources	• Pundir, S. K., and Pundir, R. *Calculus of Variation*. Pragati Prakashan (Chapter 1- Art 1.1 to 1.13, Chapter 2- Art 2.1 to 2.5, Chapter 3- Art 3.1 to 3.5) • Gelfand, I. M., and Fomin, S. V. *Calculus of Variation*. Prentice Hall.	

F.Y.M.Sc. (Mathematics)
SEMESTER-II
MTH-DSC-525: Practical course on Analytic Number Theory and Complex Analysis

Total Hours: 30 **Credits: 2**

Course objectives	• To know concept of arithmetic functions. • To study congruences and quadratic residues. • To know the concepts of primitive root theory. • To make student aware of advances in complex analysis. • To know Mobius transformation and conformal mappings.	
Course outcomes	After successful completion of this course, students are expected to: • understand the concept of Mobius function μ(n), The Euler totientfunctionφ(n), Mangolt function Λ(n), Liouvilles function λ(n), Thedivisor function σ(n), Bell series. • explain Residue classes, Lagrange's theorem and its applications, Polynomial congruences with prime power moduli. • learn Quadratic residues, existence and non-existence of primitive roots. • understand theMobius transformation, conformal mappings and complex integration in various forms.	
Practical No.	**Content**	**Hours**
1	**Practical No. 1:Arithmetic Functions** 1. Show that $\boldsymbol{\phi(n)}$ is always even for all $\boldsymbol{n \geq 3}$. 2. Find $\boldsymbol{\phi(gcd\{\phi(315), \phi(525)\})}$. 3. Find all $\boldsymbol{n}$ such that $\boldsymbol{\phi(n) = 24}$. 4. Find $\boldsymbol{\mu(5819)}$. 5. Prove that $\sum_{d\mid n} \boldsymbol{\mu(d)} = \begin{cases} \boldsymbol{1\ if n = 1} \\ \boldsymbol{0\ if n > 1} \end{cases}$. 6. Show that $\boldsymbol{\mu}$ and $\boldsymbol{u}$ are Dirichilet's inverses of each other. 7. Findi)$\boldsymbol{\phi^{-1}(36)}$ ii) $\boldsymbol{\phi^{-1}(8671)}$. 8. Show that $\boldsymbol{\lambda^{-1}(n) = \|\mu(n)\|}$for all $\boldsymbol{n} \in \mathbb{N}$. 9. Find i)$\boldsymbol{\sigma_2(12)}$ ii)$\boldsymbol{(\sigma_2)^{-1}(18)}$. 10. Prove that $\boldsymbol{\Lambda(n) = -\sum_{d\mid n} \mu(d) \log(d)}$.	**4**
2	1. With usual notations, show that $\boldsymbol{\Lambda * u = u'}$. 2. Let $\boldsymbol{f}$ be a multiplicative function. Prove that $\sum_{d\mid n} \boldsymbol{\mu(d)f(d)} = \prod_{\substack{p\mid n \\ p is prime}} \boldsymbol{(1 - f(p))}$. 3. Find the Bell series of Mobius function $\boldsymbol{\mu}$. 4. Find the Bell series of $\boldsymbol{\sigma_\alpha}$. 5. Let $\boldsymbol{f}$, $\boldsymbol{g}$ be arithmetic functions. Prove that $\boldsymbol{(f * g)_p(x) = f_p(x)g_p(x)}$ for all prime number $\boldsymbol{p}$. 6. Let $\boldsymbol{f}$, $\boldsymbol{g}$ be arithmetic functions. Prove that $\boldsymbol{(f * g)' = f' * g + f * g'}$	**4**

	7. Let $\boldsymbol{f}$, $\boldsymbol{g}$ be arithmetic functions. Prove that (i)$(\boldsymbol{f}+\boldsymbol{g})' = \boldsymbol{f}' + \boldsymbol{g}'$. (ii) $(\boldsymbol{f}^{-1})' = -\boldsymbol{f}' * (\boldsymbol{f} * \boldsymbol{f})^{-1}$, where $\boldsymbol{f}(\mathbf{1}) \neq \mathbf{0}$. 8. Find all incongruent $\boldsymbol{modulo}\ \mathbf{6}$ solutions of $\mathbf{4}\boldsymbol{x} \equiv \mathbf{2}(\boldsymbol{mod}\ \mathbf{6})$ 9. Find all incongruent $\boldsymbol{modulo}\ \mathbf{8}$ solutions of $\mathbf{12}\boldsymbol{x} \equiv \mathbf{4}(\boldsymbol{mod}\ \mathbf{8})$ 10. Solve the linear congruence $\mathbf{5}\boldsymbol{x} \equiv \mathbf{20}(\boldsymbol{mod}\ \mathbf{15})$.	
3	**Practical No. 3: Quadratic Residues and Quadratic Reciprocity Law** 1. True or False? Justify. i) Is $\{\mathbf{376}, \mathbf{531}, -\mathbf{530}, \mathbf{629}\}$ a CRS modulo 4? ii) Is $\{-\mathbf{31}, \mathbf{59}, \mathbf{405}, -\mathbf{1}\}$ an RRS modulo 8? 2. Find the remainder when $(\mathbf{26})! \times (\mathbf{23})^{\mathbf{26}}$ is divided by $\mathbf{29}$. 3. Let $\boldsymbol{p}$ be an odd prime and $\boldsymbol{a}$ be an integer such that $\boldsymbol{p} \nmid \boldsymbol{a}$. Show that $\boldsymbol{a}^{\mathbf{2}}$ is a quadratic residue of $\boldsymbol{p}$. 4. Let $\boldsymbol{p}$ be any odd prime and $\boldsymbol{a} \equiv \boldsymbol{b}\ (\boldsymbol{modp})$. Show that $\boldsymbol{a}$ is a quadratic residue of $\boldsymbol{p}$ if and only if $\boldsymbol{b}$ is a quadratic residue of $\boldsymbol{p}$. 5. Let $\boldsymbol{p} = \prod_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{k}} (\boldsymbol{p_i})^{\boldsymbol{\alpha_i}}$ be the prime power factorization of an odd integer ≥ 3. Show that $(\boldsymbol{n}\|\boldsymbol{p}) = \mathbf{0}$ if and only if $(\boldsymbol{n}, \boldsymbol{p}) > \mathbf{1}$. 6. Determine whetheri)$-\mathbf{78}$ is a quadratic residue or quadratic non-residue of the prime number 137. ii) $-\mathbf{104}$ is a quadratic residue or quadratic non-residue of the prime number 229. 7. For an odd prime $\boldsymbol{p}$, prove that $(\mathbf{2}\|\boldsymbol{p}) = (-\mathbf{1})^{\frac{p^2-1}{8}} = \begin{cases} \mathbf{1}\ \boldsymbol{ifp} \equiv \pm\mathbf{1}(\boldsymbol{mod}\ \mathbf{8}) \\ -\mathbf{1}\ \boldsymbol{ifp} \equiv \pm\mathbf{3}(\boldsymbol{mod}\ \mathbf{8}) \end{cases}$. 8. Find the value of $(\mathbf{143}\|\mathbf{37})$ and $(\mathbf{271}\|\mathbf{187})$. 9. Show that $\mathbf{5}$ is a quadratic residue of an odd prime $\boldsymbol{p}$ if $\boldsymbol{p} \equiv \pm\mathbf{1}(\boldsymbol{mod}\ \mathbf{10})$. 10. Show that $\mathbf{5}$ is a quadratic non-residue of an odd prime $\boldsymbol{p}$ if $\boldsymbol{p} \equiv \pm\mathbf{3}(\boldsymbol{mod}\ \mathbf{10})$.	**4**
4	**Practical No. 4: Primitive Roots** 1. For $\boldsymbol{k} \in \mathbb{N}$, find i)$\boldsymbol{exp_5}(\mathbf{3}^{\boldsymbol{k}})$ ii) $\boldsymbol{exp_7}(\mathbf{4}^{\boldsymbol{k}})$. 2. Prove that $\mathbf{2}^{\boldsymbol{n}}$ has no primitive root, for all $\boldsymbol{n} \geq \mathbf{3}$. 3. Prove that $\mathbf{2}^{\boldsymbol{n}}\boldsymbol{k}$ has no primitive root, where $\boldsymbol{n} \geq \mathbf{2}$ and $\boldsymbol{k}$ is an odd integer $> \mathbf{1}$. 4. Find a primitive root of i) 54 ii) $\mathbf{686}$. 5. Find the number of incongruent $\boldsymbol{mod}\ \mathbf{107}$ primitive roots of $\mathbf{107}$. 6. Find the number of incongruent $\boldsymbol{mod}\ \mathbf{113}$ primitive roots of $\mathbf{113}$. 7. Show that 187 and $\mathbf{221}$ has no primitive root. 8. If $\boldsymbol{g}$ is an odd number and $\boldsymbol{n} \geq \mathbf{3}$, then show that $\boldsymbol{g}^{\mathbf{2}^{\boldsymbol{n}-\mathbf{2}}} \equiv \mathbf{1}\ (\boldsymbol{mod}\mathbf{2}^{\boldsymbol{n}})$. 9. Let $\boldsymbol{g}$ be a primitive root of $\boldsymbol{m}$ and $(\boldsymbol{a}, \boldsymbol{m}) = \mathbf{1}$. If $\boldsymbol{a} \equiv \boldsymbol{b}\ (\boldsymbol{modm})$,	**3**

	then prove that $ind_g(a) = ind_g(b)$. 10. Let g , h be primitive roots of m and $(a, m) = 1 = (b, m)$. Prove that $ind_g(ab) \equiv \left(ind_g(a) + ind_g(b)\right) (mod\phi(m))$.	
5	**Practical No. 5: Radius of convergence and Analytic functions** 1. Find the radius of convergence of function $f(z) = \sum_{n=0}^{\infty} a^n z^n$. 2. Find the radius of convergence of function $f(z) = \sum_{n=0}^{\infty} a^{n^2} z^n$. 3. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} z^{n(n+1)}$. 4. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} k^n z^n$. 5. Find the radius of convergence of the power series e^z. 6. Find the period of e^z. 7. Show that $f(z) = \|z\|^2 = x^2 + y^2$ has derivative only at origin. 8. Show that the function $u = \log(x^2 + y^2)^{\frac{1}{2}}$ is hormonic on $\mathbb{C} - \{0\}$. 9. Show that the function $u = \tan^{-1}(\frac{y}{x})$ is hormonic on $\mathbb{C} - \{0\}$. 10. Show that $\lim_{n\to\infty} n^{\frac{1}{n}} = 1$.	**4**
6	**Practical No. 6: Fixed point and Cross ratio** 1. Find the fixed point of the function $s(z) = \frac{z-1}{z+1}$ and verify it. 2. Find the fixed point of the function $s(z) = \frac{z+1}{z-1}$. 3. Find the fixed point of a dilation, translation and inversion if $s(z) = \frac{az+b}{cz+d}$. 4. Find the fixed point of translation function $f(z) = z - 3$. 5. Find the fixed point of translation function $f(z) = \bar{z}$. 6. Evaluate the cross ratio $(7 + i, 1, 0, \infty)$. 7. Evaluate the cross ratio $(2, 1 - i, 1, 1 + i)$. 8. Evaluate the cross ratio $(0, 1, i - 1)$. 9. Evaluate the cross ratio $(i - 1, \infty, i + 1, 0)$. 10. If $T_z = \frac{az+b}{cz+d}$ find z_2, z_3, z_4 (in terms of a, b, c, d) such that $T_z = (z_1, z_2, z_3, z_4)$.	**4**
7	**Practical No. 7: Cauchy integral theorem and Cauchy integral formula** 1. Evaluate by Cauchy integral formula $\int_c \frac{e^{2z}}{(z+1)^4} dz$ where c is $\|z\| = 3$. 2. Evaluate by Cauchy integral formula $\int_c \frac{z^2+2}{(z-1)(z-2)} dz$ where c is $\|z - 1\| = 2$. 3. Evaluate by Cauchy integral formula $\int_c \frac{z+1}{z^2+1} dz$ where c is $\|z - i\| = 3$.	**4**

	4. Evaluate by Cauchy integral formula $\int_c \frac{e^z}{(z-1)(z-4)} dz$ where c :$\|z\| = 2$. 5. Evaluate $\int_c \frac{z^2-z+1}{z-1} dz$ where c is $\|z\| = 1$. 6. Evaluate $\int_c \frac{z^2+1}{z(2z+1)} dz$ where c is $\|z\| = 1$. 7. Evaluate the integral $\int_\gamma \frac{e^{iz}}{z^2} dz$ where $(t) = e^{it}$, $0 \le t \le 2\pi$. 8. Evaluate the integral $\int_\gamma \frac{sinz}{z^3} dz$ where $(t) = e^{it}$, $0 \le t \le 2\pi$. 9. Evaluate the integral $\int_\gamma \frac{sinz}{z} dz$ where $(t) = e^{it}$, $0 \le t \le 2\pi$. 10. Evaluate the integral $\int_\gamma \frac{dz}{z^2+1} dz$ where $(t) = 2e^{it}$, $0 \le t \le 2\pi$.	
8	**Practical No. 8: Singularity and Cauchy residues theorem** 1. Locate and classify the singularity of the function $f(z) = \frac{1}{\sin(\frac{\pi}{2})}$. 2. Locate and classify the singularity of the function $f(z) = \frac{\cot(\pi z)}{(z-2)^2}$. 3. Locate and classify the singularity of the function $f(z) = \frac{sinz}{z}$. 4. Locate and classify the singularity of the function $f(z) = \frac{sinz}{z^2}$. 5. Locate and classify the singularity of the function $f(z) = e^{\frac{1}{z}}$. 6. Using Cauchy residue theorem evaluate $\int_c \frac{z^2}{(z-1)^3(z-2)} dz$ where c is $\|z\| = 2.5$. 7. Using Cauchy residue theorem evaluate $\int_c \frac{zcosz}{(z-2)^2} dz$ where c is $\|z-1\| = 1$. 8. Using Cauchy residue theorem evaluate $\int_c ze^{\frac{1}{z}} dz$ where c is $\|z\| = 1$. 9. Using Cauchy residue theorem evaluate $\int_c \frac{1-2z}{2(1-z)(z-2)} dz$ where c is $\|z\| = 1.5$. 10. Using Cauchy residue theorem evaluate $\int_c \frac{1}{sinhz} dz$ where c is $\|z\| = 4$.	**3**
Study resources	• Apostol, T. M. (1972). *Introduction to Analytic Number Theory* (Student ed.). Springer International. (Sec. 2.1 - 2.19, Sec. 5.2, 5.4, 5.5, 5.6, 5.9, 5.10, Sec. 9.1 to 9.8, Sec. 10.1to 10.10). • Burton, D. M. (1980). *Elementary Number Theory*. Universal Book Stall. • Silverman, J. H. (2001). *A Friendly Introduction to Number Theory* (2nd ed.). Prentice Hall. • Niven, I., Zuckerman, H. S., and Montgomery, H. L. (1991). *An introduction to the theory of numbers* (5th ed.). John Wiley and sons. • Conway, J. B. (1995). *Functions of One Complex variable* (2nd ed.). Springer Int. • Ponnusammy, S., & Silverman, H. (2006). *Complex Variables with Applications*. Birkhauser. • Ahlfors, L. V. (1996). *Complex Analysis*. McGraw-Hill Book Co.	

F.Y.M.Sc. (Mathematics)
SEMESTER-II
MTH-OJT-526: On Job Training / Internship

Total Hours: 120 **Credits: 4**

Course objectives	• To provide the students with actual work experience • To make aware prescribe standards and guidelines at work • To develop the employability of participating student • To avail an opportunities to eventually acquire job experiences
Course outcomes	After successful completion of this course, students are expected to: • Get actual work experience with office and virtual exposure to various management styles, technical, industrial, and procedural systems • Acquaint the knowledge related to working hours, work protocols and guidelines • Understand the roles and responsibilities of employee as well as team work • Justify job experiences that match their potentials, skills, and competencies
	Internship: An internship is a professional learning experience that offers meaningful, practical work related to a student's field of study or career interest. An internship gives a student the opportunity for career exploration and development, and to learn new skills. **On the job training:** On the job training is a form of training provided at the workplace. During the training, employees are familiarized with the working environment they will become part of. Employees also get a hands-on experience using machinery, equipment, tools, materials, etc.

Skills imparted:

The curriculum is designed to inculcate basic principles of mathematical methods and analysis to apply in various fields of scientific research. The curriculum contains a wide variety of mathematical topics like topology, linear algebra, differential equations, numerical analysis, transformations, operations research, fluid mechanics, functional analysis and mathematical methods. Further the following skills are developed on successful completion:

- critical thinking
- problem solving
- analytical thinking
- quantitative reasoning
- ability to manipulate precise and intricate ideas
- construct logical arguments and expose illogical arguments
- time management
- teamwork
- independence

Job opportunity:

The designed curriculum offers job opportunities like:

- mathematics teacher
- Scientist
- Programmer
- Software professional
- Banker
- Accountant.
- Actuary
- Data analyst
- Engineer
- Investment manager
- Research leading to Ph. D. degree
- Self entrepreneurship