

Mathematics in Ancient India

Mathematics on the Indian subcontinent has a rich and long history going back over 5,000 years and thrived for centuries before advances were made in Europe. Its influence spread to China, Southeast Asia, the Middle East, and Europe. Apart from introducing the concept of zero, Indian mathematicians made seminal contributions to the study of geometry, arithmetic, binary mathematics, the notion of negative numbers, algebra, trigonometry, and calculus among other areas. The decimal place value system that is employed worldwide today was first developed in India. Classical period (400–1600) This period is often known as the golden age of Indian Mathematics. This period saw mathematicians such as Aryabhata, Varahamihira, Brahmagupta, Bhaskara I, Mahavira, Bhaskara II, Madhava of Sangamagrama and Nilakantha Somayaji give broader and clearer shape to many branches of mathematics.

Concept of Zero:

'Zero' was discovered by an anonymous Indian. The concept of zero in India holds significant historical and mathematical importance. The numeral zero, as we understand it today, is an integral part of the decimal numeral system, and it was developed in ancient India.

1. Early Development: India's earliest recorded use of a decimal system and zero can be traced back to the Indus Valley Civilization, which existed around 2500-1500 BCE. However, the concept of zero as a number with its own value and significance started to evolve later.

2. Brahmagupta's Contributions: One of the key figures in the development of the concept of zero in India was the mathematician Brahmagupta, who lived around 598-668 CE. He made significant contributions to the understanding of zero. He defined zero as a number and discussed its mathematical properties. He introduced rules for arithmetic operations involving zero, such as the addition and subtraction of zero, which are now fundamental concepts in mathematics.

3. Aryabhata's Work: Aryabhata, an Indian mathematician and astronomer who lived around 476-550 CE, also played a role in zero development. In his work, Aryabhata used a symbol to represent zero and recognized its importance in mathematical calculations.

4. Spread to the Islamic World and Europe: The concept of zero and the decimal numeral system with its place-value notation spread from India to the Islamic world. Islamic scholars like Al-Khwarizmi further developed these ideas, and they eventually made their way to Europe during the Middle Ages.

5. Importance in Modern Mathematics: Zero is now considered one of the most fundamental concepts in mathematics. It serves as a placeholder in the decimal system, enabling us to represent numbers of varying magnitudes efficiently. It is also crucial in algebra, calculus, and many other branches of mathematics.

The concept of zero in India has a long and rich history, with mathematicians like Brahmagupta and Aryabhata making significant contributions to its development.

Concept of Pi:

The concept of π (pi), which represents the mathematical constant approximately equal to 3.14159, has a long history in India, as well as in other ancient civilizations. In India, pi was known and calculated by mathematicians and astronomers in various ways over the centuries.

1. Ancient Texts: Indian mathematicians and astronomers in ancient times made references to pi in their texts. For example, the "Shulba Sutras," which are a collection of ancient Indian texts dealing with geometry and construction, include approximations of the value of pi

2. Aryabhata: Aryabhata, an Indian mathematician and astronomer who lived around 476-550 CE, calculated an approximate value of pi as 3.1416 in his work "Aryabhatiya." He used a geometric method to estimate pi, which was later refined by other Indian mathematicians.

3. Bhaskara I: Bhaskara I, who lived in the 7th century CE, also provided an approximation of pi in his work "Mahabhaskariya." He used a different method involving a polygon with 384 sides to estimate pi more accurately.

4. Madhava: The mathematician Madhava of Sangamagrama, a prominent figure of this school, developed series expansions for trigonometric functions, including pi. Madhava's work predated European developments in calculus and infinite series by several centuries.

It's important to note that while Indian mathematicians made notable contributions to the understanding and approximation of pi, the concept of pi and its accurate calculation were also pursued in various other ancient civilizations, such as Babylon, Egypt, and Greece. The approximate value of π is $\frac{22}{7}$ or 3.14159265359.

Number System:

India has a rich history of number systems, with significant contributions to the development of various numeral systems. One of the most influential contributions is the decimal numeral system, which is now the most widely used number system globally. Overview:

1. Indus Valley Civilization: The earliest known evidence of a numerical system in the Indian subcontinent can be traced back to the Indus Valley Civilization (around 2500-1500 BCE). Archaeological findings indicate the use of symbols for counting and recording quantities.

2. Brahmi Numerals: The Brahmi script, dating back to around the 3rd century BCE, played a significant role in the development of numeral systems in India. It had symbols for numbers, and these symbols are considered some of the earliest written representations of numerical values in India.

3. Kharosthi numeral system: (3rd century BCE to 3rd century CE) The Kharosthi numeral system was a positional decimal system similar to the more widely known Indian numeral system. It used symbols to represent numbers, and the position of a symbol in a number determined its value, just as in the Indian numeral system. The Kharosthi numeral symbols included various lines and curves to represent different numbers. They had symbols for numbers from 1 to 9, and a symbol for zero, making it a decimal system. The Kharosthi script and numerals have been found on inscriptions, coins, and other ancient artifacts.

4. Decimal System: The most significant contribution of India to the world of mathematics is the decimal numeral system. This system is based on the number ten and uses ten symbols (0-9) to represent all possible numbers. The concept of zero (0) as a placeholder and as a number itself was developed in India. The decimal system, with its place-value notation, simplifies arithmetic calculations and is the foundation of modern mathematics.

5. Invention of Zero: The concept of zero as both a placeholder and a number is attributed to Indian mathematicians. The use of zero revolutionized mathematics and made complex calculations much more manageable.

6. Influence on Other Cultures: The decimal system with zero travelled from India to various parts of the world, including the Islamic world and, eventually, Europe. Islamic scholars like Al-Khwarizmi played a crucial role in transmitting Indian mathematical knowledge to the West, where it became the basis for modern mathematics.

7. Other Numeral Systems: India also had regional numeral systems. e.g. The Gupta numeral system was used in the Gupta Empire (around 4th to 6th century CE). These systems had unique symbols for numbers but were gradually replaced by the more efficient decimal system.

In summary, India's contribution to the development of the decimal numeral system with zero is one of its most significant intellectual achievements. This system has had a profound and lasting impact on mathematics and has become an integral part of global numerical and mathematical notation.

Pythagoras Theorem:

The Pythagorean theorem, which states that in a right-angled triangle, the square of the length of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides, is a fundamental theorem in geometry. While the theorem is named after the ancient Greek mathematician Pythagoras, similar geometric principles were known and applied in various cultures, including India, long before Pythagoras' time.

In India, there is evidence to suggest that knowledge of geometric principles related to the Pythagorean theorem was present in ancient times:

1. Sulba Sutras: The Sulba Sutras are a collection of ancient Indian texts that date back to around 800 BCE to 200 BCE. These texts primarily deal with geometry and provide rules for the construction of altars for religious ceremonies. Some of these rules involve right-angled triangles and the application of what we now recognize as the Pythagorean theorem. While they may not state the theorem explicitly, they demonstrate an understanding of the relationship between the sides of right-angled triangles.

2. Baudhayana and Apastamba: Baudhayana, an ancient Indian mathematician, and Apastamba, a sage and mathematician. They provided geometric methods for constructing right angles and calculating diagonal measurements that align with the principles of the Pythagorean theorem.

3. Aryabhata: In his work "Aryabhatiya," he discussed the relationships between the lengths of chords and the radii of circles, which are related to the Pythagorean theorem.

While these Indian mathematicians and texts demonstrated knowledge of geometric principles related to the Pythagorean theorem, it's important to note that the theorem is named after Pythagoras because he is traditionally credited with providing the first formal proof of the theorem in ancient Greece.

Vedic Mathematics:

Vedic Mathematics is a system of mathematics that claims to be based on the ancient Indian scriptures known as the Vedas. It is often attributed to a scholar named Swami Bharati Krishna Tirtha, who published a book titled "Vedic Mathematics" in 1965, where he introduced and popularized this system. However, it's important to note that the legitimacy and historical accuracy of Vedic Mathematics as a distinct mathematical system is a topic of debate among mathematicians and scholars. Key features and claims associated with Vedic Mathematics:

1. Sutras: Vedic Mathematics is said to be based on a set of 16 Sutras (aphorisms) and 13 Upa-sutras (sub-sutras) extracted from the Vedas. These Sutras are claimed to provide efficient and unconventional methods for performing various mathematical operations, such as addition, subtraction, multiplication, division, square roots, and cube roots.

2. Mental Calculation: Vedic Mathematics emphasizes mental calculation and claims to offer techniques that allow individuals to perform complex mathematical computations in their heads quickly. Proponents of Vedic Mathematics argue that these techniques can lead to faster and more efficient calculations compared to traditional methods.

3. General Applicability: Vedic Mathematics is said to be applicable to a wide range of mathematical problems and is not limited to specific areas of mathematics. Proponents claim that it can be used in various fields, including arithmetic, algebra, geometry, and calculus.

4. Controversy and Criticism: While Vedic Mathematics has gained popularity in some educational circles and has been the subject of books and workshops, it has also faced criticism and skepticism from mainstream mathematicians and educators. Critics argue that many of the techniques promoted as Vedic Mathematics are not significantly different from or more efficient than established mathematical methods.

5. Educational Use: Vedic Mathematics has been introduced as a teaching tool in some schools and educational programs. It can help students develop mental math skills and improve their speed and accuracy in calculations.

In summary, Vedic Mathematics is a system of mathematical techniques and methods that claim to be based on ancient Indian scriptures. While it has gained some popularity and is promoted as a way to improve mental calculation skills.

Aryabhata:

Aryabhata was an ancient Indian mathematician and astronomer who lived in the late **5th century CE**. He is one of the most significant and influential mathematicians and scientists in the history of India. Key contributions and information about Aryabhata:

1. Aryabhatiya: Aryabhata is best known for his major work, the "Aryabhatiya," a Sanskrit text consisting of 121 verses. This work covers a wide range of topics in mathematics and astronomy. In the "Aryabhatiya," Aryabhata introduced innovative mathematical concepts and techniques.

2. Value of Pi (π): Aryabhata approximated the value of pi (π) as 3.1416, which was a significant improvement over earlier estimates. His work provided a more accurate understanding of the ratio of the circumference of a circle to its diameter.

3. Trigonometry: Aryabhata made significant contributions to trigonometry. He introduced trigonometric concepts and the sine function, which he referred to as "jya." His trigonometric tables and methods laid the foundation for further developments in Indian mathematics.

4. Astronomy: Aryabhata's work in astronomy included calculations related to the positions of planets, the lunar eclipse, and the solar eclipse. He proposed a heliocentric model of the solar system in which he correctly stated that the Earth revolves around the Sun. This heliocentric idea was well ahead of its time and predated similar ideas in Europe by many centuries.

5. Number System: Aryabhata made contributions to the Indian number system, including the use of a place-value notation system and the concept of zero as a placeholder.

It's important to note that Aryabhata is often referred to as Aryabhata I, as there were other mathematicians and astronomers with the same or similar names who lived in later centuries. Aryabhata's contributions to mathematics, astronomy, and trigonometry remain significant, and his work is considered a cornerstone of ancient Indian science and mathematics. Aryabhata was the first of the significant mathematician-astronomers from the classical age of Indian mathematics and Indian astronomy. He treated mathematics as a different discipline.

Mahaviracharya:

Mahavira, also known as Mahaviracharya or Mahavira II, was a renowned mathematician and Jain philosopher who lived in India in the **9th century CE**. He made significant contributions to the field of mathematics, particularly in the area of number theory. Key contributions:

1. Ganita Sara Sangraha: Mahavira is best known for his work "Ganita Sara Sangraha," which translates to "Compendium of the Essence of Mathematics." This text is a comprehensive treatise on mathematics and is considered one of the most important works on Indian mathematics.

2. Number Theory: Much of Mahavira's work focused on number theory, including the study of prime numbers, factors, divisibility, and algebraic equations. He made important contributions to understanding the properties of numbers.

3. Prime Factorization: Mahavira developed a novel method for prime factorization, which involved a systematic procedure for finding the prime factors of a given number. His method was more efficient than earlier techniques and is still studied by mathematicians today.

4. Continued Fractions: He also made advancements in the study of continued fractions, which are expressions in which a number is represented as an infinite series of fractions. Mahavira's work on continued fractions was pioneering and contributed to the broader field of mathematics.

5. Jain Philosophy: In addition to his mathematical contributions, Mahavira was a Jain philosopher and scholar. Mahavira's work in mathematics was in harmony with Jain principles of intellectual pursuit and knowledge.

Mahavira's "Ganita Sara Sangraha" remains an important mathematical text, and his contributions to number theory continue to be studied and appreciated by mathematicians and scholars interested in the history of mathematics in India.

Bodhayana:

Bodhayana, also known as Bodhayana Acharya, is an ancient Indian mathematician and scholar known for his significant contributions to mathematics. He is believed to have lived around the **6th century BCE**, making him a prominent figure in the history of Indian mathematics. Bodhayana's work primarily focused on geometry and geometric principles.

One of Bodhayana's most notable contributions is his commentary on the "Brahmasphutasiddhanta," which was originally written by the renowned Indian mathematician and astronomer Brahmagupta. Bodhayana's commentary provided valuable insights and explanations for the mathematical concepts and principles presented in the "Brahmasphutasiddhanta."

In his commentary, Bodhayana elaborated on various mathematical topics, including arithmetic, algebra, and geometry. He expanded upon the geometric concepts and theorems found in Brahmagupta's work, which included ideas related to:

- 1. Geometry:** Bodhayana's commentary covered various aspects of geometry, including the properties of triangles, circles, and other geometric shapes. He explained the principles of constructing altars and geometric figures.
- 2. Algebra:** Bodhayana contributed to the understanding of algebraic concepts and equations. His commentary contained explanations of algebraic expressions and equations, demonstrating his mathematical prowess.

Bodhayana's work, along with other ancient Indian mathematical texts and commentaries, played a crucial role in the development and transmission of mathematical knowledge in ancient India. His contributions, as well as those of other Indian mathematicians and scholars, helped shape the foundation of Indian mathematics and laid the groundwork for further advancements in the field.

Bhaskaracharya:

Bhaskaracharya, also known as Bhaskara II or Bhaskaracharya II, was a renowned Indian mathematician and astronomer who lived in the **12th century CE**. He was one of the most significant mathematicians and scholars of the medieval period in India. Contributions:

- 1. Mathematics:** Bhaskaracharya made important contributions to mathematics, particularly in the areas of algebra, arithmetic, and calculus. His work "**Lilavati**" is one of his most famous mathematical treatises. In this book, he covered various mathematical topics, including number theory, geometry, and solutions to mathematical problems.

2. Astronomy: Bhaskaracharya was also an accomplished astronomer. He wrote the astronomical treatise known as the "Siddhanta Shiromani." This work is divided into four sections: Lilavati (arithmetic and geometry), Bijaganita (algebra), Grahaganita (mathematics of the planets), and Goladhyaya (spherical trigonometry). In the Grahaganita section, he made significant contributions to understanding planetary motion and eclipses.

3. Bijaganita: Bhaskaracharya's "Bijaganita" is a comprehensive work on algebra. It covers topics like solving equations, indeterminate equations, and algebraic methods for finding square roots.

4. Calendar Reform: Bhaskaracharya proposed calendar reforms and made adjustments to the Indian calendar system. His insights into the calculation of leap years and the adjustment of months contributed to the accuracy of Indian calendars.

Bhaskaracharya's works, especially "Lilavati" and "Siddhanta Shiromani," remain important sources for understanding the mathematical and astronomical knowledge of his era.

Varahamihira:

Varahamihira, also known simply as Varaha, was an ancient Indian scholar and polymath who lived during the **6th century CE**. He made significant contributions to various fields of knowledge, including astronomy, mathematics, astrology, and meteorology. Contributions:

1. Astronomy and Astrology: Varahamihira is best known for his astronomical and astrological works. His most famous work, "Brihat-Samhita" (The Great Compilation), is a comprehensive treatise that covers a wide range of topics, including astronomy, astrology, mathematics, architecture, and more. It provided valuable information about planetary positions, eclipses, and other celestial phenomena.

2. Mathematics: Varahamihira made contributions to mathematics, particularly in the context of astronomy and mathematical calculations related to calendars. He was skilled in the use of numbers and mathematical techniques for astronomical and astrological predictions.

3. Meteorology: Varahamihira also wrote about meteorology in "Brihat-Samhita," offering insights into weather patterns, forecasting, and related phenomena. His work included observations about natural disasters and atmospheric conditions.

4. Calendar Reform: He proposed reforms to the Indian calendar system to make it more accurate. His recommendations influenced the development of Indian calendars and their adjustment for leap years.

5. Jyotisha: Varahamihira was a prominent figure in the field of Jyotisha (Indian astrology). He made significant contributions to the study of horoscopes, planetary movements, and the interpretation of celestial events.

Varahamihira's "Brihat-Samhita" remains an important source of information for understanding ancient Indian knowledge systems, especially in the fields of astronomy and astrology.

Brahmagupta:

Brahmagupta was an ancient Indian mathematician and astronomer who lived during the **7th century CE**. He was one of the most influential mathematicians of his time and made significant contributions to various branches of mathematics, including algebra, number theory, and geometry. Key aspects of Brahmagupta's life and contributions:

1. Brahmasphutasiddhanta: Brahmagupta is best known for his mathematical treatise titled "Brahmasphutasiddhanta," which translates to "The Corrected Treatise of Brahma." This work covers a wide range of mathematical topics and is considered one of the most important mathematical texts from ancient India.

2. Algebra: In the "Brahmasphutasiddhanta," Brahmagupta made significant contributions to algebra. He introduced the concepts of positive and negative numbers, including zero, and discussed their properties and operations. His work laid the foundation for the systematic study of integers and rational numbers.

3. Zero: One of Brahmagupta's most famous contributions is his treatment of zero as a number with its own mathematical properties. He discussed zero's role as a placeholder in numerical notation and explained rules for arithmetic operations involving zero.

4. Astronomy: Brahmagupta was also an accomplished astronomer. His work included observations of celestial phenomena, such as planetary positions, eclipses, and the positions of stars and planets. He developed methods for calculating the positions of celestial objects.

5. Geometry: Brahmagupta made contributions to geometry as well. He studied various geometric shapes and properties, including the area of triangles and cyclic quadrilaterals. His work on geometric theorems and proofs was influential in the field.

6. Trigonometry: Brahmagupta contributed to trigonometry by developing formulas and relationships involving trigonometric functions, particularly for right-angled triangles.

Brahmagupta's works were widely respected and studied in India and were later translated into other languages, including Arabic and Persian, which contributed to the spread of his mathematical ideas to other parts of the world.

The Kerala School of Mathematics and Astronomy:

The Kerala School of Mathematics and Astronomy, also known as the Kerala School of Astronomy and Mathematics, was a prominent center for mathematical and astronomical research and scholarship that flourished in the southern Indian state of Kerala between the **14th and 16th centuries CE**. This school of thought made significant contributions to various fields of mathematics, astronomy, and related sciences.

Key aspects and contributions of the Kerala School of Mathematics and Astronomy include:

1. Trigonometry: The Kerala School is renowned for its contributions to trigonometry. The mathematicians of this school developed elaborate trigonometric theories and methods for calculating various trigonometric functions. One of their most significant achievements was the discovery of infinite series expansions for trigonometric functions like sine, cosine, and tangent, which predated European developments in calculus.

2. Calculation Techniques: The Kerala mathematicians developed innovative and efficient calculation techniques for complex mathematical and astronomical problems. These techniques included methods for the rapid calculation of planetary positions, lunar eclipses, and other celestial phenomena.

3. Astronomy: The Kerala School made notable advancements in observational and computational astronomy. They developed models for predicting planetary motions and eclipses and contributed to the understanding of celestial events.

4. Mathematical Literature: The scholars of the Kerala School produced numerous mathematical and astronomical texts. Some of the well-known texts include "Yuktibhāṣā" by Jyesthadeva, "Tantrasangraha" by Nilakantha Somayaji, and "Karana Paddhati" by Sankara Variar.

5. Calendar Reforms: The Kerala School played a role in calendar reforms. They made adjustments to the Indian calendar system, taking into account astronomical observations and calculations to ensure the accuracy of calendars and festivals.

6. Influence: While the Kerala School was primarily active in Kerala, its ideas and discoveries had a broader influence. They influenced developments in mathematical and astronomical thought in other parts of India and the world.

7. Connection to Earlier Indian Traditions: The Kerala School built upon earlier Indian mathematical and astronomical traditions, including contributions from scholars like Aryabhata, Brahmagupta, and others. They expanded upon existing knowledge and developed new methodologies.

The Kerala School's achievements in mathematics and astronomy were significant and attracted the attention of scholars and travellers from different parts of the world. Their work in trigonometry, in particular, was ahead of its time and had a substantial impact on the development of mathematics and science globally.

Dattatreya Ramachandra Kaprekar:

He was well known for "Kaprekar Constant" 6174. Take any four-digit number in which all digits are not alike. Arrange its digits in descending order and subtract from it the number formed by arranging the digits in ascending order. If this process is repeated with remainders, ultimately number 6174 is obtained, which then generates itself. Kaprekar discovered the Kaprekar constant or 6174 in 1949. Thus, starting with 1234, we have

$$4321 - 11234 = 3087, \text{ then}$$

$$8730 - 0378 = 8352, \text{ and}$$

$$8532 - 2358 = 6174.$$

Repeating from this point onward leaves the same number ($7641 - 1467 = 6174$).

Srinivasa Ramanujan

Ramanujan was born on 22 December 1887 into a Tamil Brahmin Iyengar family in Erode, in present-day Tamil Nadu. He was one of India's greatest mathematical geniuses. He made substantial contributions to the analytical theory of numbers and worked on elliptic functions, continued fractions, and infinite series. In January 1913 Ramanujan wrote to G H Hardy having seen a copy of his 1910 book *Orders of infinity*. In Ramanujan's letter to Hardy he introduced himself and his work. Srinivasa Ramanujan's work has found application in physics and engineering. G.H. Hardy invited Ramanujan to England. He developed rapidly as a mathematician under the guidance of G. H. Hardy, who brought him to Cambridge University in 1916. He published more than 340 mathematical papers spanning various mathematical areas.

- The film 'The Man who knew infinity' is based on Ramanujan's life.
- 2012 year was celebrated as National Mathematics Year by the Government of India.



Ancient Indian Mathematician and Their Salient Contributions

Sr. No.	Detail of the Work/ Mathematician	Period, Location	Salient Contributions
1	Vedic Texts	3000BCE or earlier	The earliest recorded mathematical knowledge, number system, Pythagorean type triplets; Decimal system of naming numbers, the concept of infinity.
2	Sulba-sutras- Baudhayana, Apasthamba, Katyayana and Manava Sulba-sutras	800-600 BCE	Earliest text of geometry; Approximate value of the square root of 2, and π . Exact procedures for the construction and transformations of squares, rectangles, trapezia, ect.
3	Aryabhata- Aryabhatiyam	476-550 CE; Kusumapura, near Pataliputra, Bihar	Concise verses; Algorithm for square root, cube root, Place value system; Sine table; geometry; quadratic equations; Linear indeterminate equations; Sums of squares and cubes of numbers; Planetary astronomy; Plane and spherical trigonometry.
4	Varaha Mihira- Brhat Samhita, Brhat-jataka, Panca-siddhantika	482-565 CE; Ujjain, Madhya Pradesh	Summary of five ancient siddhantas; Sine table, trigonometric identities; $\sin^2 + \cos^2$; Combinatorics; Magic squares.
5	Bhaskara I- Commentary on Aryabhatiya, Laghu-bhaskariyam and Maha-bhaskariyam.	600-800 CE; Vallabhi region, Saurashtra, Gujarat	Expanded Aryabhata's work on Integer solution for indeterminate equations; Approximate formula for the sine function, Planetary Astronomy.
6	Brahmagupta- Brahmasphuta-siddhanta, Khandakhadyaka	598-668 CE; Bhillamala in Rajasthan	Rules of arithmetic operations with zero and negative numbers, Algebra (Bijaganita); linear and quadratic indeterminate equations; Pythagorean triplets, Formula for the diagonals and area of a cyclic quadrilateral; notion of arithmetic mean.

7	Mahaviracarya- Ganita-sara-sangraha	800-870 CE; Gulbarga, Karnataka	A comprehensive, exclusive textbook on mathematics covering arithmetic-geometry-algebra, continuing the ancient Jain mathematics tradition; permutations and combinations; arithmetic and geometric series; the sum of squares and cubes of numbers in arithmetic progression.
8	Shripati- Ganita-tilaka, Siddhanta-sekhara, Dhikotidakarana, ect.	1019-1066 CE; Rohinikhanda, Maharashtra	Planetary Astronomy
9	Bhaskaracarya (Bhaskara-II)- Lilavati on arithmetic and geometry; Bijaganita on algebra; Siddhanta-siromani on astronomy; Vasabhasya on Siddhanta-siromani.	1114-1185 CE; Hailed from Bijjadavida	Canonical textbooks used all over India, Detailed explanations including Upapatti (demonstration or proof); addition formula for sine function, Surds; permutations and combinations; Solution for indeterminate equations, Ideas of calculus, including mean value theorem, planetary astronomy; construction of several instruments.
10	Kamalakara- Siddhanta-tattva-viveka	1616-1700 CE; Varanasi, Uttara Pradesh	Addition and subtraction theorems for the sine and the cosine; Sines and cosines of double, triple, etc., angles.